On the Relation Between Credit Spread Puzzles and the Equity Premium Puzzle

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Credit Spread *Level* Puzzle

- Difficult to reconcile theoretical (structural) models of credit risk with observed credit spreads. Jones Mason Rosenfeld (84), Eom, Helwege, Huang (01), Huang and Huang (03)...
  - Historical expected loss explains about 1% of CS for short maturity Aa bonds (Elton, Gruber, Agrawal, Mann (01))
  - Conditional on fitting historical expected loss structural models predict low spreads (Huang Huang 03).

$$\begin{array}{llll} \text{Baa-Treas.} \approx 32\text{bp} & \text{vs.} & \text{actual } 158 \text{ bp} \\ \text{Aaa-Treas.} \approx \ \ 1\text{bp} & \text{vs.} & \text{actual } \ \ 55 \text{ bp.} \end{array}$$

$\Rightarrow$ Credit spreads ($\sim$ average returns on investment grade bonds) seem "too high" relative to the risk they entail (*conditional on standard model*).

- Reminiscent of the 'equity premium puzzle' which concludes that historical stock returns seem 'too high' relative to their risk (*conditional on standard model*).
- N.B.: Various extensions of structural models model bankruptcy/conflicts of interest/strategic debt service do not explain Credit Spread Puzzle per se.
  - Anderson Sundaresan (96), Mella-Barral and Perraudin (96), Francois Morellec (04)...
- Other explanations aside from expected loss and risk-premium: Liquidity, Taxes?

Credit Spread *Time-Variation* Puzzle

- Benchmark model with time-varying risk-premium investigated by HH predicts Baa-Aaa spread solely driven by leverage.
  ($CS = f(lev, r, \sigma, mat) \approx$ constant for 'refreshed' Baa, Aaa rating).
- Hard to reconcile changes in credit spreads with predictions of:
  - Structural models (CDGM (2001))
    - Substantial amount of covariation among individual firm credit spreads unrelated to factors predicted by structural models.
  - Reduced form models (Berndt, Douglas, Duffie, Ferguson, Schranz (03), CDGH (03)).
    - Highly time varying and large jumps in pricing kernel at time of individual firm default (or contagion) needed to explain 'jump risk premium'.
- Historical data exhibits:
  - High Variation in Baa-Aaa credit spread ($\sim$ 72bp).
  - Countercyclical default rates (regression coefficient of four-year default rates on credit spreads has coefficient equal to 1.2)

Can we solve the Credit Spread Puzzles?

Q? Which models can raise credit spreads while matching historical expected recovery and default rates?

$$
\begin{aligned}
P &= \mathrm{E}\left[\Lambda\left(1-\mathbf{1}_{\{\tau\le T\}}L_\tau\right)\right] \\
&= \mathrm{E}\left[\Lambda\right]\mathrm{E}\left[1-\mathbf{1}_{\{\tau\le T\}}L_\tau\right] + \mathrm{Cov}\left[\Lambda\,,\left(1-\mathbf{1}_{\{\tau\le T\}}L_\tau\right)\right] \\
&= \frac{1}{R^f}\left(1-E\left[\mathbf{1}_{\{\tau\le T\}}L_\tau\right]\right) - \mathrm{Cov}\left[\Lambda\,,\mathbf{1}_{\{\tau\le T\}}L_\tau\right].
\end{aligned}
$$

- To lower prices of risky bonds (and thus raise spreads) need to:

  – increase the covariation between the pricing kernel and the default time,

  – increase the covariation between the pricing kernel and the loss given default.

⇒ Consistent with results of HH, who find that considering different structural models with the same risk-premium does not help.

- Structural models define (value triggerd) default as first passage of asset value, V_t, at some default boundary, B_t ($\sim$ liabilities):

$$\tau := \inf\{t \ : \ V_t \leq B_t\}$$

$\Rightarrow$ Basically three chanels to 'explain' credit spread puzzle:

  - increase magnitude of covariation between the pricing kernel and asset value,
  - increase covariation between the pricing kernel and the default boundary process,
  - increase covariation between the pricing kernel and the loss given default.

Relation between Credit puzzles and Equity premium puzzle?

This paper investigates whether pricing kernel models engineered to fit the equity premium can also predict magnitude and time variation of credit spreads when:

- extracting the risk-premium from equity data.
- fitting unconditional historical average loss rate.

Specifically, we consider two models of the equity risk premium:

- Campbell and Cochrane (1999) *'habit formation'* model
  - Equity premium explained by *time varying risk-aversion*: Investors become highly risk-averse in bad (i.e., low consumption/dividend) states.
- Bansal and Yaron (2004) *'long run risk'* model
  - Equity premium explained by *cash flow risk*: persistent shocks to aggregate consumption & dividend growth rates + stochastic volatility make equity more risky than in an i.i.d. world.
  - Actually, since BY model combines cash flow risks with time varying risk-premia, we distinguish three cases:
    1. Case I: growth rate risk only
    2. Case II: growth rate risk and stochastic volatility
    3. Case III: cash flow risk (I+II) and time varying risk-premia.

- These pricing kernels are then incorporated into structural models of credit risk with:

  1. Constant nominal default boundary (Black Cox (76), Longstaff Schwartz (95))
  2. Time varying (i.e., stochastic) default boundary (CDG (01)) calibrated to match historical relationship between credit spreads and future default rates.

     (Solved using Monte-Carlo Simulation with importance sampling.)

- Look at implications for:

  – Population average level of credit spread.
  – Population variance (i.e., time variation) of credit spread.
  – Covariance of credit spread and four-year default rates.
  – Fitted historical time series of spreads.
  – Correlation between spreads, consumption dynamics, and the equity premium.

Results

- None of the models can explain the level or time-variation of Aaa-Treasury spread
  - historical default rates are too small.
    (Non-default related? Risk-free benchmark?)

- With constant default boundary, models explain between 40% and 60% of Baa-Aaa spread.

  - The CC and BY I& II models explain $<$ 50% of the Baa-Aaa spread.
  - BY III explains $\sim$ 60% of the Baa-Aaa spread.
  - CC generates little variability in spreads and procyclical default rates.

- With a time-varying countercyclical default boundary CC model explains over 90% of spread and most of the time-variation.

- However, default boundary needs to be more countercyclical than historical market leverage.

Interpretation

- To raise spreads need to increase risk-neutral default probability (e.g., using stochastic volatility in BY or countercyclical boundary in CC.)

- To do this while maintaining low historical default rates, need kernel that generates high time variation in risk-premia (i.e., generates a large wedge between physical and risk-neutral probabilities).

- Results suggest that:

→ Risk-premia extracted from equity market can improve our prediction of credit spreads.

→ To reconcile bond prices and equity prices, structural model requires default boundary that is more countercyclical than leverage (liquidity?).

Outline of Paper

- Stylized facts.
- Benchmark HH model (constant coefficients).
- CC model
  - Calibration of pricing kernel to consumption and dividend data.
  - Calibration to historical default rates
  - Implication for credit spreads
- 'BY' model(s)
  - Calibration of pricing kernel to consumption and dividend data.
  - Calibration to historical default rates
  - Implication for credit spreads
- Conclusions

Data and Benchmark Model

- Stylized facts:
  - Baa-Aaa spreads are high on average (122 bp) and very volatile (72 bp std dev).
  - Default rates are low on average (4-year default rate Baa $= 1.4\%$) and volatile.
  - Default rates are countercyclical in that the correlation between spreads and 4-year future default rates is positive (≈ 0.5).
  - Typical Baa firm leverage countercyclical.

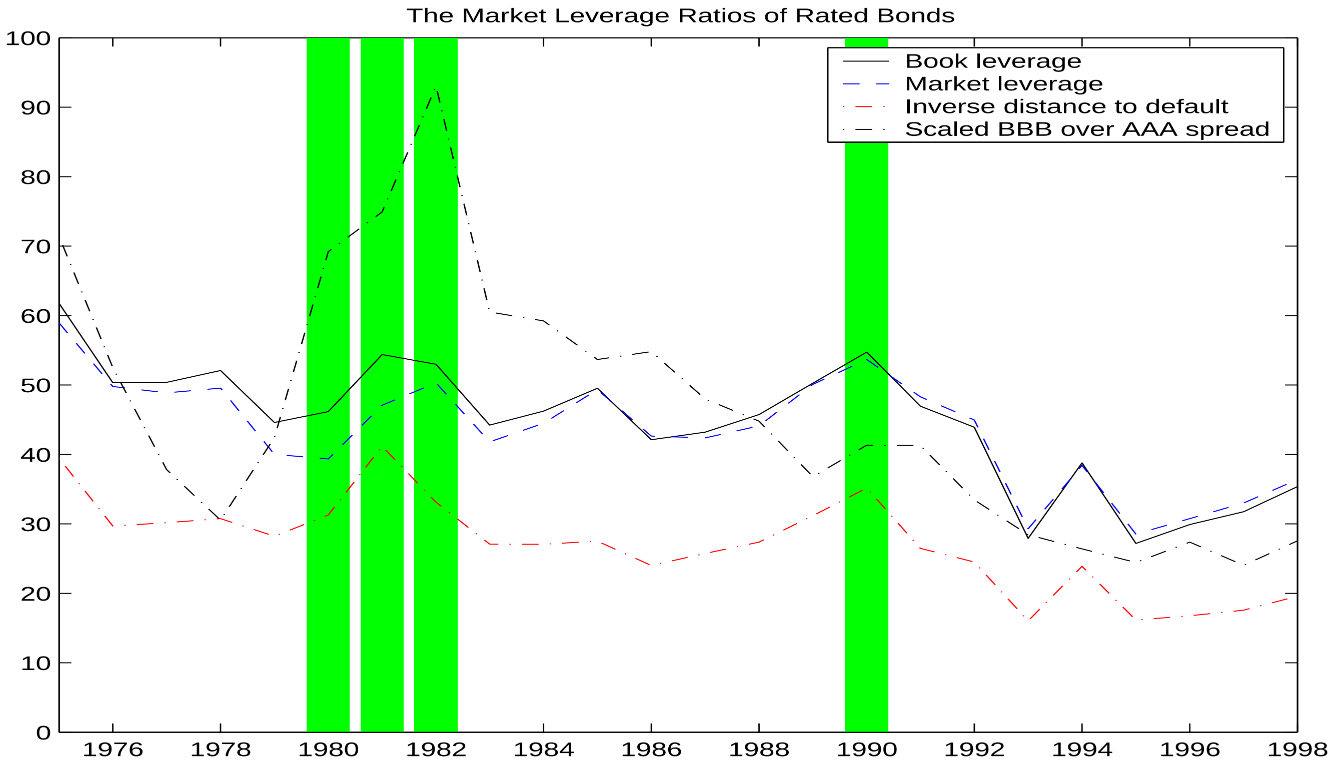


Figure 1: Time series of leverage for Baa rated firms.

Panel A:	Summary statistics			
Variables	Mean	Std.	Min.	Max.
4-year default probability (%)	1.39	1.08	0.00	4.00
10-year Baadefault probability	5.00	2.34	1.17	9.70
Aaa over Treasury spread	0.75	0.43	0.21	1.84
Baa over Treasury spread	1.90	0.98	0.66	5.52
Baa over Aaa spread	1.22	0.72	0.37	4.20
P/D ratio	23.40	7.22	11.48	49.43
Book leverage of Baa	0.46	0.09	0.27	0.62
Market leverage of Baa	0.44	0.08	0.29	0.59
Inverse of the DD of Baa	0.28	0.07	0.16	0.42

Panel B:	Correlation matrix of some benchmark variables						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
P/D ratio (1)	1.00						
Consumption innovation (2)	0.11	1.00					
	0.35						
Baa over Aaa spread (3)	-0.43	-0.33	1.00				
	0.00	0.00					
4-year default probability (4)	-0.27	0.02	0.47	1.00			
	0.16	0.93	0.01				
Book leverage of Baa (5)	-0.70	-0.29	0.55	0.12	1.00		
	0.00	0.17	0.01	0.58			
Market leverage of Baa (6)	-0.61	-0.23	0.47	0.07	0.96	1.00	
	0.00	0.27	0.02	0.74	0.00		
Inverse of the DD of Baa (7)	-0.68	-0.39	0.58	0.16	0.96	0.86	1.00
	0.00	0.06	0.00	0.46	0.00	0.00	

Panel C:	Regressions of default probability on Baa - Aaa spread		
Dependent variable	intercept	Baa - Aaa spread	Adj R sq
4-year default probability	0.02	1.20	0.19
	0.36	4.09	
10-year default probability	-0.01	4.04	0.43
	1.13	4.19	

Table 1: Summary statistics. The statistics of different variables cover different periods. The Baa over Aaa spread and the P/D ratio cover 1919-1997 period. The 4-year (10-year) default probability covers 1970-1998 (1970-1992) period. Book leverage is defined as the ratio of book debt to (book debt + market equity). Market leverage is defined as the ratio of market debt to (market debt + market equity). The inverse of the distance to default (DD) is defined as the ratio of (0.5*long term book debt + short term book debt) to (market debt + market equity). In panel B, the first (second) row is the correlation (p-value). The correlation statistics use the maximum common sample size between two series. In Panel C the the first row is the OLS regression coefficients. On the second row Newey-West t-statistics are reported. 4 lags are chosen for 4-year default probability and 10 lags are chosen for 10-year default probability.

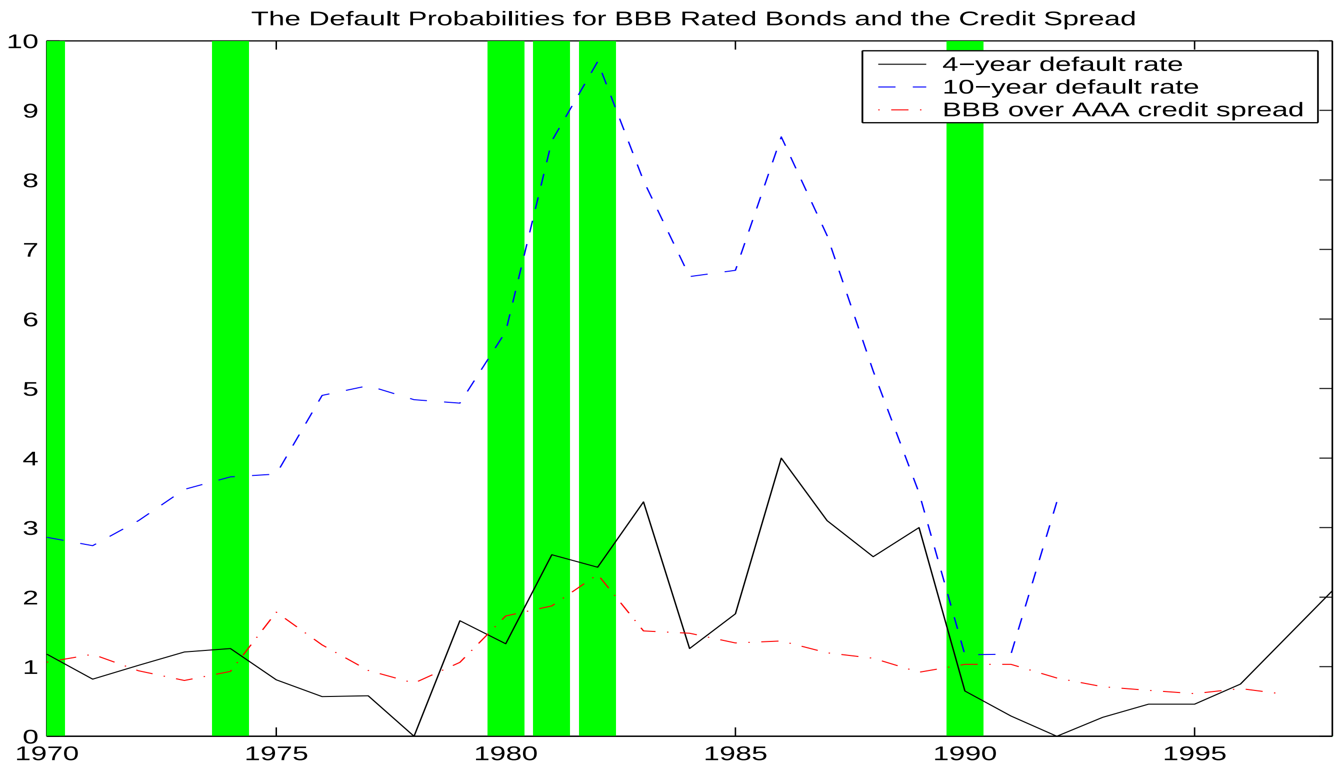

The Default Probabilities for BBB Rated Bonds and the Credit Spread
4−year default rate
10−year default rate
BBB over AAA credit spread
0
1
2
3
4
5
6
7
8
9
10
1970
1975
1980
1985
1990
1995

Figure 2: .

Benchmark model:

– Constant Default Boundary (Merton (1976) Longstaff and Schwartz (1995))

– Market portfolio dynamics:

$$\frac{dV(t)}{V(t)} = (\theta + r - \delta)\,dt + \sigma_V\,dZ_V(t)$$

– All parameters:

  * r: risk-free rate
  * θ: market risk-premium
  * δ: payout ratio
  * σ_V: market volatility.

  calibrated to fit unconditional average market risk-premium, and volatility, average price-dividend ratio, average nominal risk-free rate.

– Assume that typical Baa firm has firm value process given by:

$$\frac{dP(t)}{P(t)} = \frac{dV(t)}{V(t)} + \sigma_{\text{idio}}\,dz_{\text{idio}}(t) \tag{1}$$

– Assume default is triggered at $\tau = \inf\{t\,:\,P_t \leq B_t\}$

Benchmark model:

- Calibration:

  1. Nominal default boundary set to a constant (60% of average leverage) (HH).
  2. Recovery set equal to average historical recovery (51.31%) (HH).
  3. $\sigma_{\text{idio}} = 0.203$ chosen to fit unconditional average Baa default rate.

  $\Rightarrow$ Obtain credit Spread: $CS(\frac{P_t}{B_t}, r, \sigma, T)$ function of distance to default ($\sim$ leverage).

  4. To compute variability of spread use unconditional mean and standard deviation of observed Baa leverage.

- Results: The model fails in (at least) three respects:

  – The average spread is too low ($46bp$).

  – Conditional on an initial spread the volatility of the spread is zero.

  – Total variation in spreads (due to leverage) is too low ($38bp$).

P def prob	P Std dev	Q def prob	Q std dev	Average Spread	Std Dev. Spread	Reg Coef
0.0141	0.013	0.0383	0.03	0.0046	0.0038	3.44
(0.0001)		(0.0002)				

A remark on estimating expected first passage time

- Combine idea from 'importance sampling' with Girsanov's theorem to accelerate simulation
  1. Since under P firm value drift is high, default events occur very rarely. Need to simulate a very large number of path ω_i, $i = 1, \ldots, n$ to estimate accurately:
  $$\mathrm{E}[\mathbf{1}_{\{\tau \le T\}}] \approx \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\{\tau(\omega_i) \le T\}}$$
  2. Instead, may simulate the process:
  $$\frac{dP(t)}{P(t)} = (\theta + r - \delta - \lambda \sigma_v)\, dt + \sigma_v\, dZ_V^\lambda(t) + \sigma_{\text{idio}}\, dz_{\text{idio}}(t)$$
  where $Z_V^\lambda(t) = Z_V(t) + \lambda dt$ is a Brownian motion under $P^\lambda \sim P$.
  3. An unbiased estimate of the default probability is:
  $$\mathrm{E}[\mathbf{1}_{\{\tau \le T\}}] = \mathrm{E}^\lambda[\frac{dP}{dP^\lambda} \mathbf{1}_{\{\tau \le T\}}] \approx \frac{1}{n} \sum_{i=1}^{n} e^{-\frac{\lambda^2}{2}T + \lambda z_T^\lambda(\omega_i)} \mathbf{1}_{\{\tau(\omega_i) \le T\}}$$
  4. We can further choose the drift adjustment λ to minimize the variance of the estimator.

Campbell and Cochrane 'countercyclical risk aversion' Model

- CC specifies Pricing Kernel with following dynamics:

$$\Lambda_t = e^{-\alpha t} \left(C - \hat{C}\right)^{-\gamma} \equiv e^{-\alpha t} e^{-\gamma s} e^{-\gamma c},$$

- log consumption (c) and log-surplus/consumption ratio ($s = \log \frac{C-\hat{C}}{C}$) dynamics:

$$\begin{aligned} dc(t) &= g\,dt + \sigma\,dZ(t) \\ ds(t) &= \kappa(\overline{s} - s(t))dt + \sigma \left[\frac{1}{\overline{S}}\sqrt{1 - 2(s - \overline{s})} - 1\right] dZ(t) \end{aligned}$$

- Any claim to cash flows D_t valued by: $\boxed{V_0 = \int_0^\infty \mathrm{E}\left[\frac{\Lambda_t}{\Lambda_0} D_t\right] dt}$.

- CC set $\overline{S} = \sigma\sqrt{\frac{\gamma}{\kappa}}$ ($s_{max} \equiv \overline{s} + \frac{1}{2}\left(1 - \overline{S}^2\right)$) to obtain a constant real rate of interest

$$r_f = \alpha + \gamma g - \frac{1}{2}\gamma\kappa. \tag{2}$$

- Calibration: CC choose parameters to match postwar data of $g = 1.88\%$, $\sigma = 1.24\%$, $r_f = 0.94\%$.

- Left with two free parameters (γ, κ) which they choose to match Sharpe ratio (0.43), and the serial correlation of log P/D ratio.

	Original CC		**Our estimates:** $\gamma = 1.95$		**Our estimates:** $\gamma = 6$		**Historical Data**	
Statistics	C-claim	D-claim	C-claim	D-claim	C-claim	D-claim	Postwar data	Long data
E[dc]	1.88	1.82	1.88	1.82	1.88	2.03	1.89	1.72
sigma(dc)	1.24	9.01	1.24	9.01	1.24	9.01	1.22	3.32
E[rf]	0.94	0.94	0.94	0.94	0.94	0.94	0.94	2.92
E(r-rf)/sigma(r-rf)	0.43	0.32	0.43	0.25	0.94	0.46	0.43	0.22
E(r-rf)	6.71	6.57	3.82	3.82	5.29	5.93	6.69	3.9
sigma(r-rf)	15.64	20.33	8.89	15.21	5.63	12.97	15.73	17.96
exp[E(p-d)]	18.11	18.51	35.04	34.53	23.03	20.72	24.7	21.16
sigma(p-d)	0.28	0.3	0.15	0.17	0.08	0.1	0.26	0.27

Table 2: Means and standard deviations of simulated and historical data.

$\Rightarrow$ Remarkable 'out-of-sample predictions' of CC are (i) average risk-premium, (ii) standard deviation of returns, (iii) average P/D ratio.

- Our results show that CC model generates expected returns and standard deviations of only about half historical levels while average P/D ratio about twice the historical levels.

- Fortunately, with slightly different calibration ($\gamma = 6$ vs. 2) results for *dividend* (vs. consumption) claim are pretty good.

– Implications of CC model for Credit Spreads?

* CC model predicts that market portfolio has dynamics:

$$\frac{dV(t)}{V(t)} = \Big(\theta(s_t) + r - \delta(s_t)\Big)\ dt + \sigma(s_t)\, dz_V(t). \quad (3)$$

with endogenous *stochastic* $\theta(s)$, $\delta(s)$, $\sigma(s)$

* As in benchmark case assume that typical Baa firm value is:

$$\frac{dP(t)}{P(t)} = \frac{dV(t)}{V(t)} + \sigma_{idio}\, dz_{idio}(t). \quad (4)$$

* Assume default occurs at first passage time of firm value at *real* default boundary.
* Assume recovery of 51.31% of nominal at date of default.

– For **constant** nominal default boundary (set equal to 60% of average leverage) find:

* Average spread (**53 ± 10**)**bp,** vs. historical (122 ± 72)bp.
* **Procyclical** default rates (because risk-premia are countercyclical).

$s(0)$	Steady State Distribution	Baa-Treasury Spread	Q-Default Rate	P-Default Rate
-3.31	0.01	0.0144	0.064	0.010
-2.93	0.06	0.0062	0.054	0.011
-2.56	0.28	0.0059	0.051	0.012
-2.19	0.59	0.0053	0.046	0.015
-1.82	0.07	0.0041	0.036	0.023

– With **countercyclical** *nominal* default boundary set equal to

$$B^{nom}_{def}(s_t) = .4328 - 1.6 * e^{s_t}. \quad (5)$$

1. Implies reasonable distribution of bankruptcy costs ($B^{nom} \in [0.22, 0.4328]$).
2. Constant default boundary is just as *ad hoc*.
3. Chosen to roughly match regression coefficient of default rates on credit spreads.

we find:

* Average spread **(110 ± 42)bp**, vs. historical (122 ± 72)bp.

– However, repeating the above for Aaa bonds, CC predicts a spread of only ≈ 5bp.

$s(0)$	Steady State Distribution	Baa-treasury Spread	Q-default Rate	P-default Rate
-3.31	0.01	0.0239	0.1969	0.0270
-2.93	0.06	0.0201	0.1682	0.0229
-2.56	0.28	0.0157	0.1329	0.0189
-2.19	0.59	0.0095	0.0820	0.0124
-1.82	0.07	0.0025	0.0222	0.0069

Table 4: Model generated Baa state-dependent credit spreads

s(0)	Steady State Distribution	Aaa-Treasury Spread	Q-Default Rate	P-Default Rate+-
-3.31	0.01	0.0010	0.0086	0.0007
-2.93	0.06	0.0007	0.0072	0.0005
-2.56	0.28	0.0006	0.0057	0.0005
-2.19	0.59	0.0004	0.0035	0.0003
-1.82	0.07	0.0001	0.0010	0.0003

Table 5: Model generated Aaa state-dependent credit spreads

Can countercylical default boundary be explained by leverage?

– We regress the market leverage ratio (MLV) of Baa rated bonds on the exponential surplus consumption ratio:

$$MLV_{Baa}(s) = 0.5210 - 0.6164 * e^{s}. \qquad (6)$$

– We use this as a default boundary and obtain:

  * Low spreads (60 bp average BAA spread).
  * Procylical default rates.

– Suggests that leverage alone is not sufficient.

Bansal-Yaron 'Long Run Risks' Model

- The BY model generates the equity premium via cash flow risk as opposed to time varying risk-aversion. It adds stochastic volatility and consumption growth:

$$
\begin{aligned}
dc_t &= (\mu + x_t)dt + (v_t + \bar{v})\, dZ_c(t) \\
dd_t &= (\mu_d + \phi x_t)dt + \sigma_d (v_t + \bar{v})\, dZ_c(t) \\
dx_t &= -\kappa x_t\, dt + \sigma_x (v_t + \bar{v})\, dZ_x(t) \\
dv_t &= -\nu v_t\, dt + \sigma_v\, dZ_v(t)
\end{aligned}
$$

where c, d are the log consumption and dividend process respectively.
The pricing kernel is approximated by:

$$
\begin{aligned}
\frac{d\Lambda_t}{\Lambda_t} &= -r_t dt - (\lambda_{c0} + \lambda_{c1} v_t) dZ_c(t) - (\lambda_{v0} + \lambda_{v1} v_t) dZ_v(t) - (\lambda_{x0} + \lambda_{x1} v_t) dZ_x(t) \\
r_t &= \alpha_0 + \alpha_x x_t
\end{aligned}
$$

- All parameters are calibrated following BY to match the dividend claim return characteristics.

– Results are as follows:

	Our BY		**Original BY** $\gamma = 10$		**Our estimates CC:** $\gamma = 6$		**Historical Data**	
Statistics	C-claim	D-claim	C-claim	D-claim	C-claim	D-claim	Postwar data	Long data
E[dc]	1.8	1.8	1.8	1.8	1.88	2.03	1.89	1.72
sigma(dc)	2.7	12.16	2.7	12.16	1.24	9.01	1.22	3.32
E[rf]	0.94	0.94	0.93	0.93	0.94	0.94	0.94	2.92
sigma[rf]	0.97	0.97	0.57	0.57	0.	0.	0.97	?
E(r-rf)/sigma(r-rf)		0.40	NA	0.37	0.94	0.46	0.43	0.22
E(r-rf)		6.7	NA	6.84	5.29	5.93	6.69	3.9
sigma(r-rf)		13.0	NA	18	5.63	12.97	15.73	17.96
exp[E(p-d)]		24.36	NA	19.98	23.03	20.72	24.7	21.16
sigma(p-d)		0.24	NA	0.21	0.08	0.1	0.26	0.27
sigma(kernel)		0.56	0.73	0.73				

Table 6: Means and standard deviations of simulated and historical data.

– **Implications for Credit spreads?**

* Set $\sigma_{idio} = 0.233$ to match historical average Baa four year default rates.
* Consider three cases to distinguish cash flow risk from time varying risk-premia:
  · Case I: growth rate risk (i.e., we set the volatility and risk-premia to be constant: $\sigma_v = 0, v_t = 0$ and $\lambda_{j1} = 0\ ,\ j = c, v, x$).
  · Case II: growth rate and volatility risk (i.e., we set the risk-premia to be constant: $\lambda_{j1} = 0\ ,\ j = c, v, x$)).
  · Case III: growth rate and volatility risk as well as time varying risk-premia.

CASE I				
Stochastic growth - Constant volatility and risk-premia				
P def prob	Q def prob	Average Spread	Std Dev. Spread	Reg Coef
0.0144	0.043	0.0053	0.0009	3.019
(0.0005)	(0.001)			
CASE II				
Stochastic growth and volatility - Constant risk-premia				
P def prob	Q def prob	Average Spread	Std Dev. Spread	Reg Coef
0.0144	0.0501	0.0062	0.0023	2.269
(0.0006)	(0.0015)			
CASE III				
Stochastic growth and volatility - Time varying risk-premia				
P def prob	Q def prob	Average Spread	Std Dev. Spread	Reg Coef
0.0135	0.062	0.0078	0.0036	0.9
(0.0009)	(0.0018)			

Table 7: Estimated values of P and Q default probabilities as well as the unconditional mean and variance of the credit spread for four year to maturity Baa firms. Standard errors of estimates are in parenthesis. Parameters of the typical Baa firm are as defined above for the dividend claim with added 23.3% idiosyncratic volatility. The spread is simulated within a structural model which assumes a constant nominal default boundary at 60% of the average Baa leverage ratio ($K = 0.6 * 0.4328$. Upon default bond recover constant fraction of face value corresponding to average historical Baa recovery rate 51.31%. All quantities are in basis points. Simulations are run with 50000 runs for each price estimation (conditional on state), with standard antithetic variance reduction.

∗ The model generates

1. Case I generates spread similar to benchmark (only difference is time-varying payout ratio).
2. Case II somewhat larger spreads due to increased Q-measure default probability and countercyclical default rates due to countercyclical volatility.
3. Case III substantially increases spreads and decreases covariance between spreads and default rates.

   Time varying risk-premia linked to volatility at the same time introduce more risk-neutral defaults in high volatility states which are more expensive thus shifting probability mass from good to bad states.

⇒ Shows necessity of highly time-varying risk-premia to reconcile spread puzzles.

∗ Introducing time varying default boundary not likely to improve performance: trade-off between average default probability and co-variance between default probability and spreads.

Conclusion - Future Work

– We argue that to explain credit spread *level* and *time-variation* puzzles need to focus on different models of pricing kernel.

– We compare two models of pricing kernel that are engineered to explain the Equity premium puzzle using very different mechanisms:

1. Time varying risk-aversion generating countercyclical risk-premia (CC),
2. Time varying stochastic volatility/drift generating long-run cash-flow risk (BY).

– The CC model can explain level and time variation in credit spreads, provided it allows for a countercyclical default barrier (else generates procyclical default rates).

– The BY model generates too low an average level of spreads.

– Results suggests that to reconcile equity returns with credit spreads need:

  * high time-variation in risk-premia (highly skewed pricing kernel).
  * countercyclical default boundary (more so than leverage).

– Future Work:

1. Theoretical model of covariation between default boundary and business cycle?
   * Story: more costly to raise funds in bad states than in good states when firms need cash to avoid bankruptcy (liquidity?).
   * Consistent with CDGM in that variation in default boundary constitutes a 'common factor' across bond market.
   * Related to literature on countercyclical leverage ratios and financing constraints (Korajzcyk and Levy (2001), Gertler and Gilchrist (1994))
2. Combining habit formation pricing kernel of CC with stochastic volatility: killing two birds with one stone?
   * Improve fit of CC model with respect to postwar return/consumption data.
   * Improve credit spread prediction of CC model.
3. Relationship between spreads and risk-premia:
   * Are spreads driven by idiosyncratic risks (Campbell and Taksler (2002))?
   * Can we use spreads as proxy for unobservable equity premium (Jagannathan and Wang (1996))?

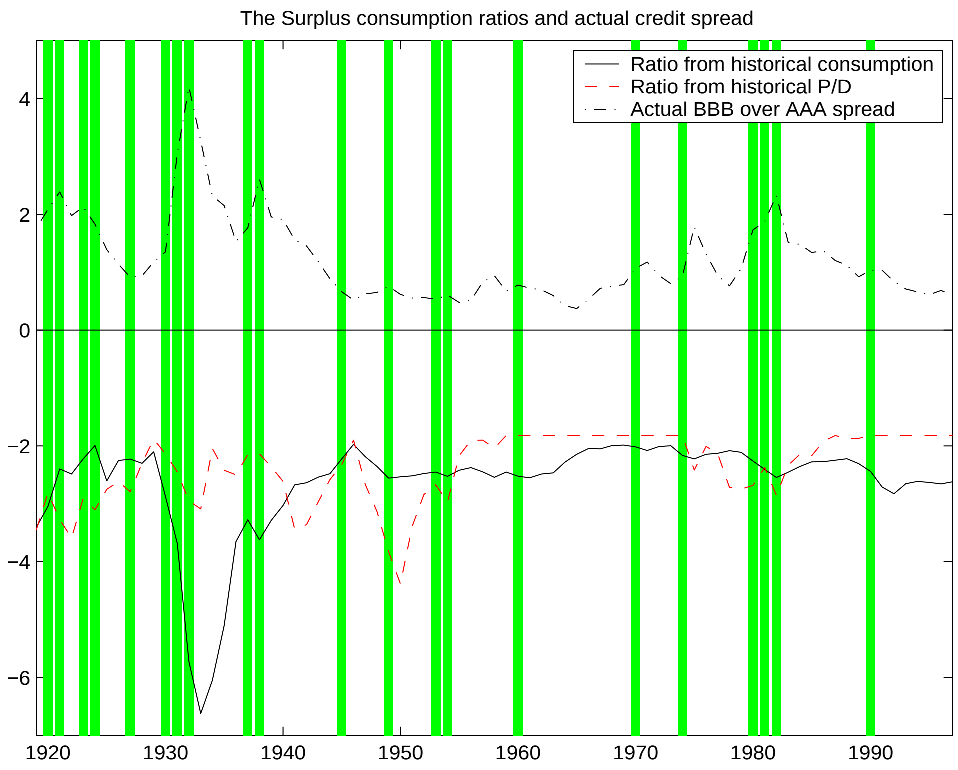


Figure 3: Time series of surplus consumption ratio of the CC model extracted using two different approaches: First, we use consumption data alone. Second we use P/D ratio.

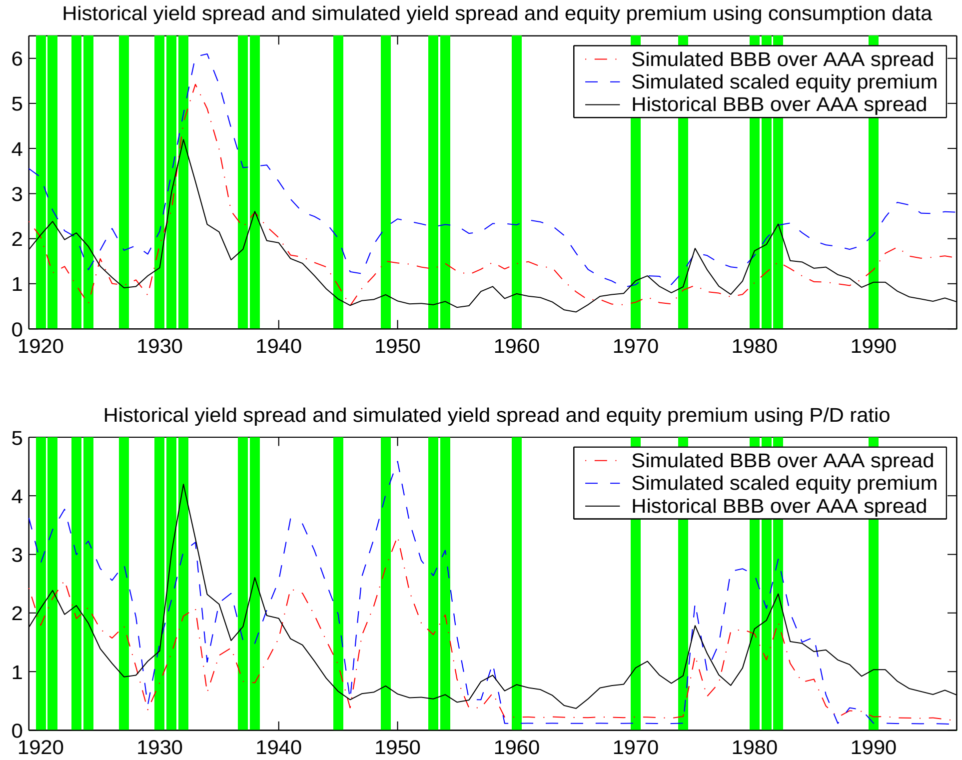


Figure 4: Time series of fitted credit spread, fitted equity premium and actual spread using two different approaches to extract the state (consumption surplus ratio) in the CC model. First, we use consumption data alone. Second we use P/D ratio.

– Our results suggests:

  * The historical level and variation of credit spreads can be matched in the CC model where the stochastic component is the variation in aggregate consumption

⇒ credit spread is likely to be mostly driven by systematic risk (and not idiosyncratic risk).

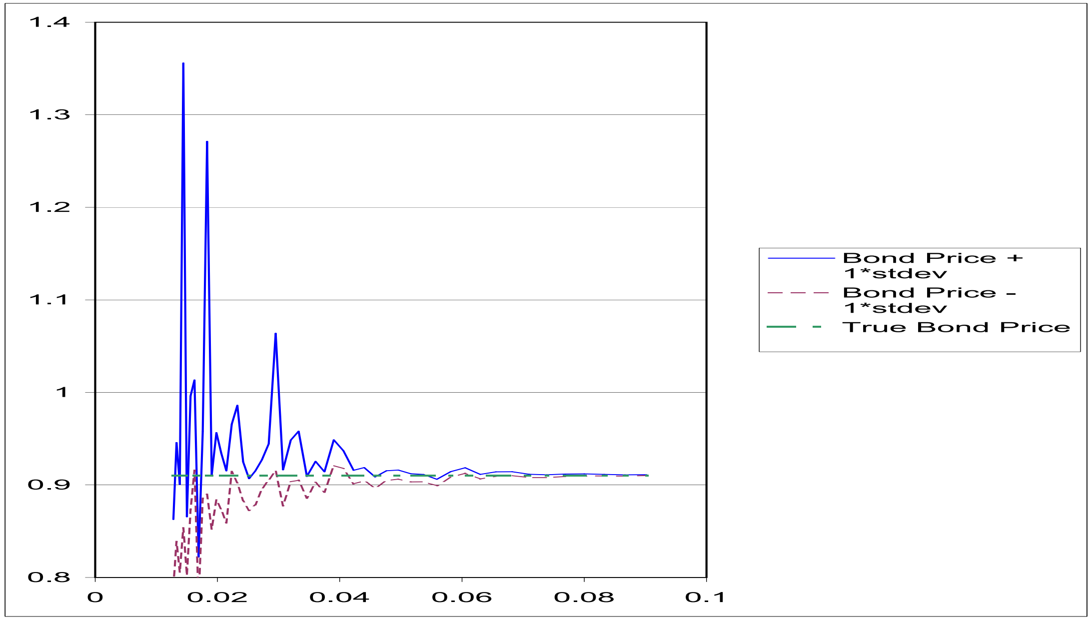
  * Historical credit spread seems to be (have been?) a good proxy for unobservable equity premium (at least until 1993).
  * One state variable model of CC does not capture jointly times series of consumption and price dividend ratio.

- Intuition for why our results differ from CC, similar to ‘importance sampling’ issue for simulating default events.
- In CC model, tail events happen with very low probability: difficult to simulate prices accurately.
- Example: Try to estimate the risk-free bond price,

$$\mathrm{E}[\frac{\Lambda_T}{\Lambda_0}] = \frac{1}{n}\sum_{i=1}^{n}\frac{\Lambda_T(\omega_i)}{\Lambda_0}$$

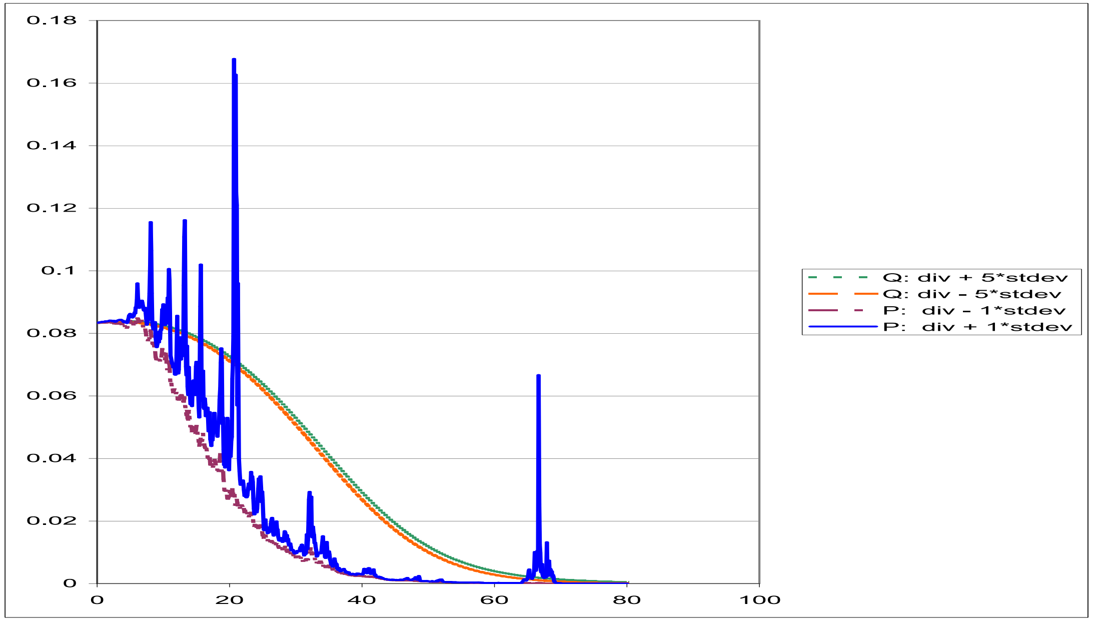
  We know the correct answer in closed form since the interest rate is constant in the model:

$$\mathrm{E}[\frac{\Lambda_T}{\Lambda_0}] = e^{-r_f T}$$



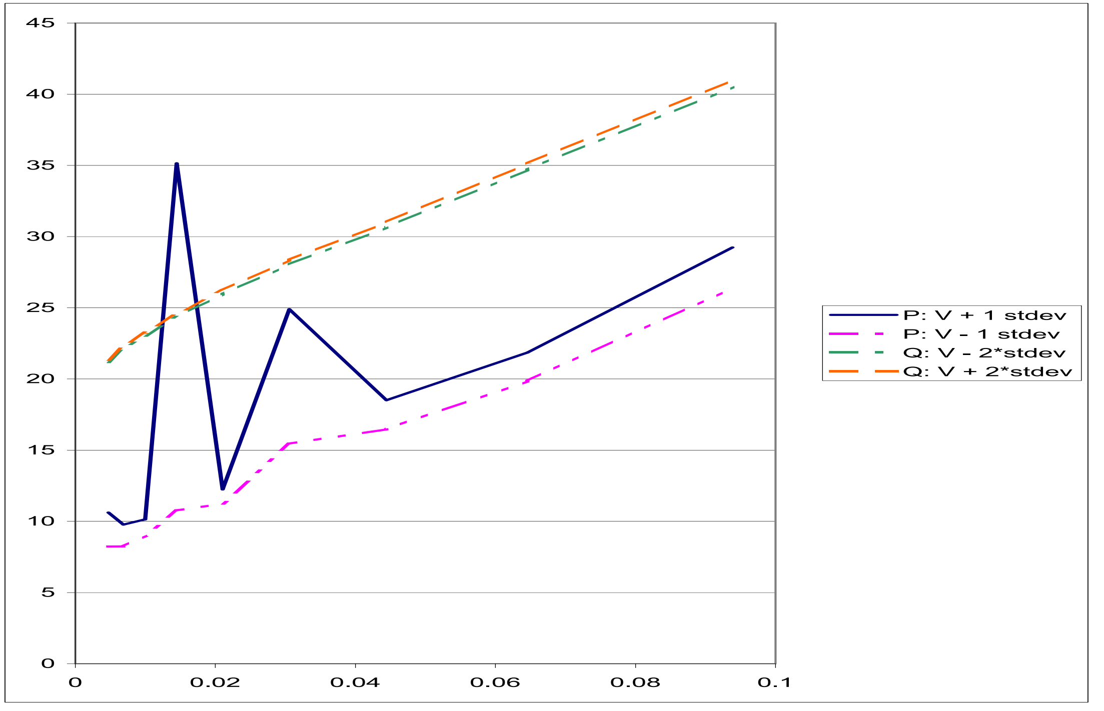
P-estimation for the value of the one-period riskless bond. The three curves are the true value, and the estimated value plus or minus one standard error. Parameter values are $g = 0.0189$, $\gamma = 2$, $\sigma = 0.015$, $\kappa = 0.138457$, $r = 0.0094$ and $\Delta t = 10$. 100,000 sample paths are used for the P-measure estimates.

- Results are worse for the claim to individual dividends (also known in closed-form).



Estimation of the value of the individual consumption claims using Monte Carlo methods under both the P and Q-measures. Parameter values are $g = 0.0189$, $\gamma = 2$, $\sigma = 0.015$, $\kappa = 0.138457$, and $r = 0.0094$. 100,000 sample paths are used for the P-measure estimates, whereas only 10,000 sample paths are used for the Q-measure estimates.

- However, simulating under the risk-neutral measure (analogous to ‘importance sampling’ idea) seems to improve results.



Estimation of the price-consumption ratio using Monte Carlo methods under both the P and Q-measures. Parameter values are $g = 0.0189$, $\gamma = 2$, $\sigma = 0.015$, $\kappa = 0.138457$, and $r = 0.0094$. 100,000 sample paths are used for the P-measure estimates, whereas only 10,000 sample paths are used for the Q-measure estimates.

A note on exchange economy prices with recursive utility

- A few recent papers (BY, Campbell et alii.) solve for asset prices (or optimal portfolio) in models where representative agent has recursive utility using Campbell-Shiller Log-linearization.
- Instead we (CDG 2005) propose improvement to their approximation.
- Consider simple Channel I dynamics:

$$\begin{aligned} dc_t &= (\mu_c + x(t))\, dt + \sigma_c\, dz_c(t) \\ dx_t &= -\kappa_x x\, dt + \sigma_x\, dz_x(t), \end{aligned}$$

- Representative agent has recursive utility:

$$J(t) = \mathrm{E}_t[\textstyle\int_t^\infty f(c_s, J_s)ds]$$

  where the 'normalized' aggregator function:

$$f(c, J) = \frac{\beta u_\rho(c)}{((1-\gamma)J)^{1/\theta - 1}} - \beta\theta J$$

  where $\theta = \frac{1-\gamma}{1-\rho}$.

- The pricing kernel in this economy is simply:

$$\Pi(t) = e^{\int_0^t f_J(c_s, J_s)ds} f_c(c_t, J_t) \tag{7}$$

- We obtain an explicit solution for the value function (and the pricing kernel):

$$J(t) = u_\gamma(c_t)(\beta I(x_t))^\theta$$

where $I(x)$ is equilibrium price-consumption ratio. It solves a non-linear ODE:

$$I\left((1-\gamma)(\mu_c + x) + (1-\gamma)^2\frac{\sigma_c^2}{2} - \beta\theta\right) + \theta(\frac{\sigma_x^2}{2} I_{xx} - \kappa_x x I_x) + \theta(\theta-1)\frac{(I_x)^2}{I}\frac{\sigma_x^2}{2} = -\theta$$

- Continuous time equivalent of Campbell-Shiller log-linearization is to guess that $I(x) = e^{A+Bx}$ and 'replace' the RHS by an exponential function:

$$f(x) = (n_0 + n_1 x)e^{A+Bx} \approx -\theta$$

where n_0, n_1 are chosen 'locally' so that $f(\mathrm{E}[x]) = -\theta$, and $f'(\mathrm{E}[x]) = 0$.

- Instead, we propose an approximation that is global in the sense that it uses information from the unconditional distribution of the state variables ($\sim$ 'Feynman-Kac'). Choose a parametric function $f(x;\Theta)$ such that:

$$\min_\Theta \quad \mathrm{E}[(f(x_t;\Theta) + \theta)^2] \text{ subject to}$$

$$f(x) = I\left((1-\gamma)(\mu_c + x) + (1-\gamma)^2\frac{\sigma_c^2}{2} - \beta\theta\right) + \theta(\frac{\sigma_x^2}{2} I_{xx} - \kappa_x x I_x) + \theta(\theta-1)\frac{(I_x)^2}{I}\frac{\sigma_x^2}{2}$$

- Essentially treat ODE as constraint and choose Θ in 'global' sense.
- For case, where $\gamma = \rho$ (Constant relative risk-aversion) we can compare both approximation (CS vs. CDG) vs. the exact closed form:

$$I(x) = \int_0^\infty e^{A(\tau)-B(\tau)x} d\tau$$

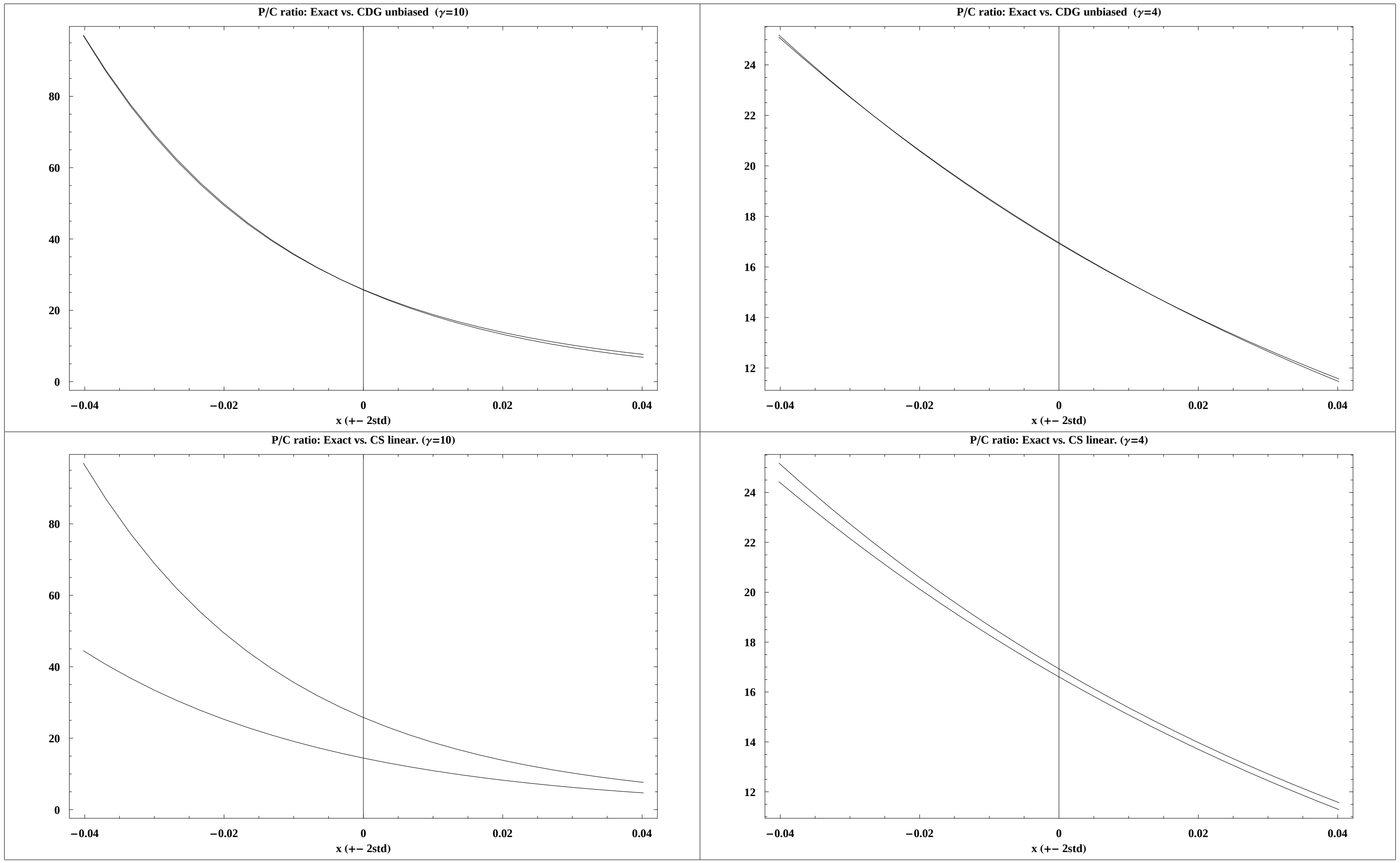


Figure 5: **Comparison of Approximations vs. Exact solution Equilibrium Price Dividend ratio**

- Our approach can be extended in several directions:
  - Refine approximation by going to higher order.
  - Works for incomplete market portfolio choice problem.
  - Can be extended to solve finite maturity portfolio choice problems.