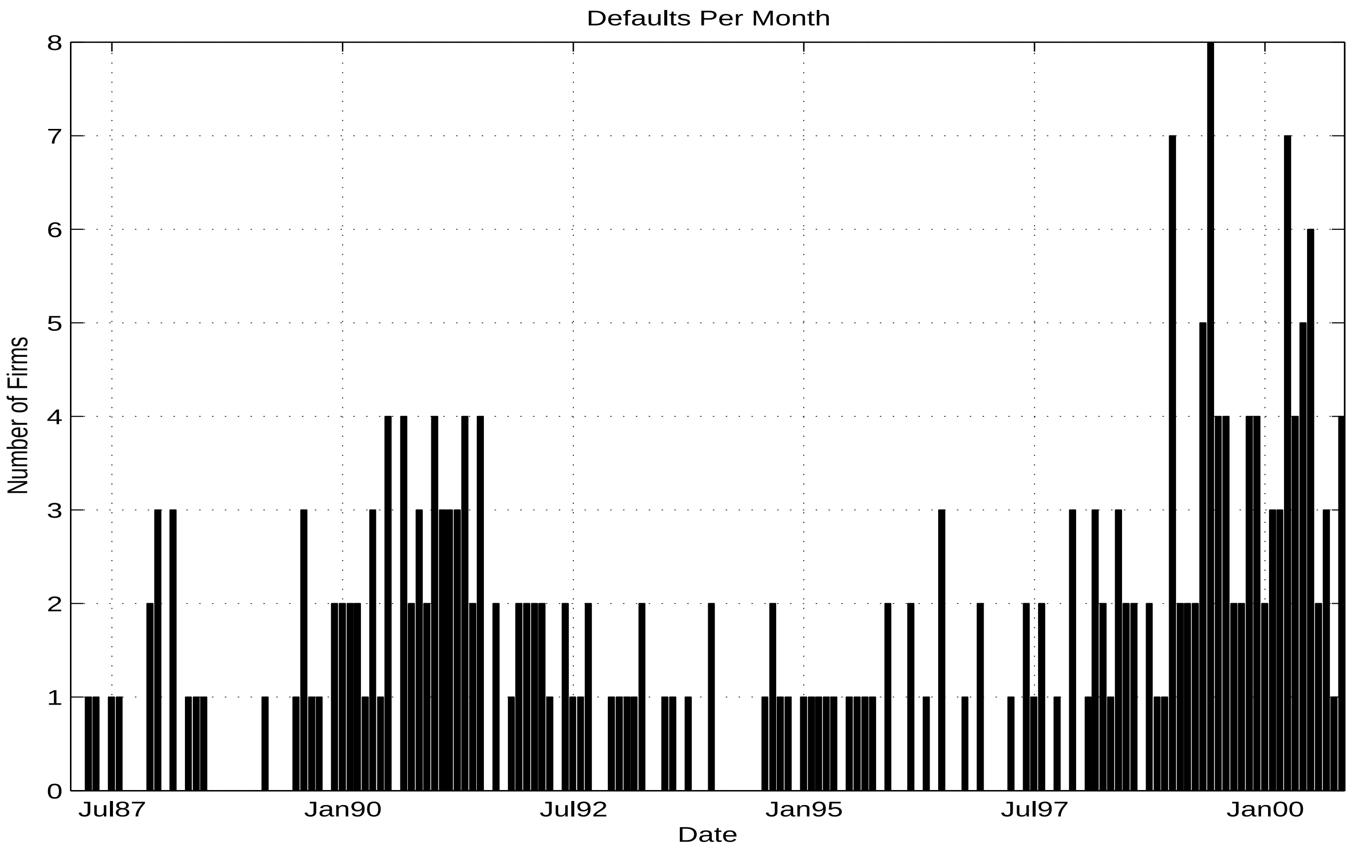
Common Failings: How Corporate Defaults are Correlated

Sanjiv R. Das
Darrell Duffie
Nikunj Kapadia
Leandro Saita

Paper:

`http://people.umass.edu/nkapadia/Research/ddk.pdf`

Defaults of Rated Firms over 1987-2000



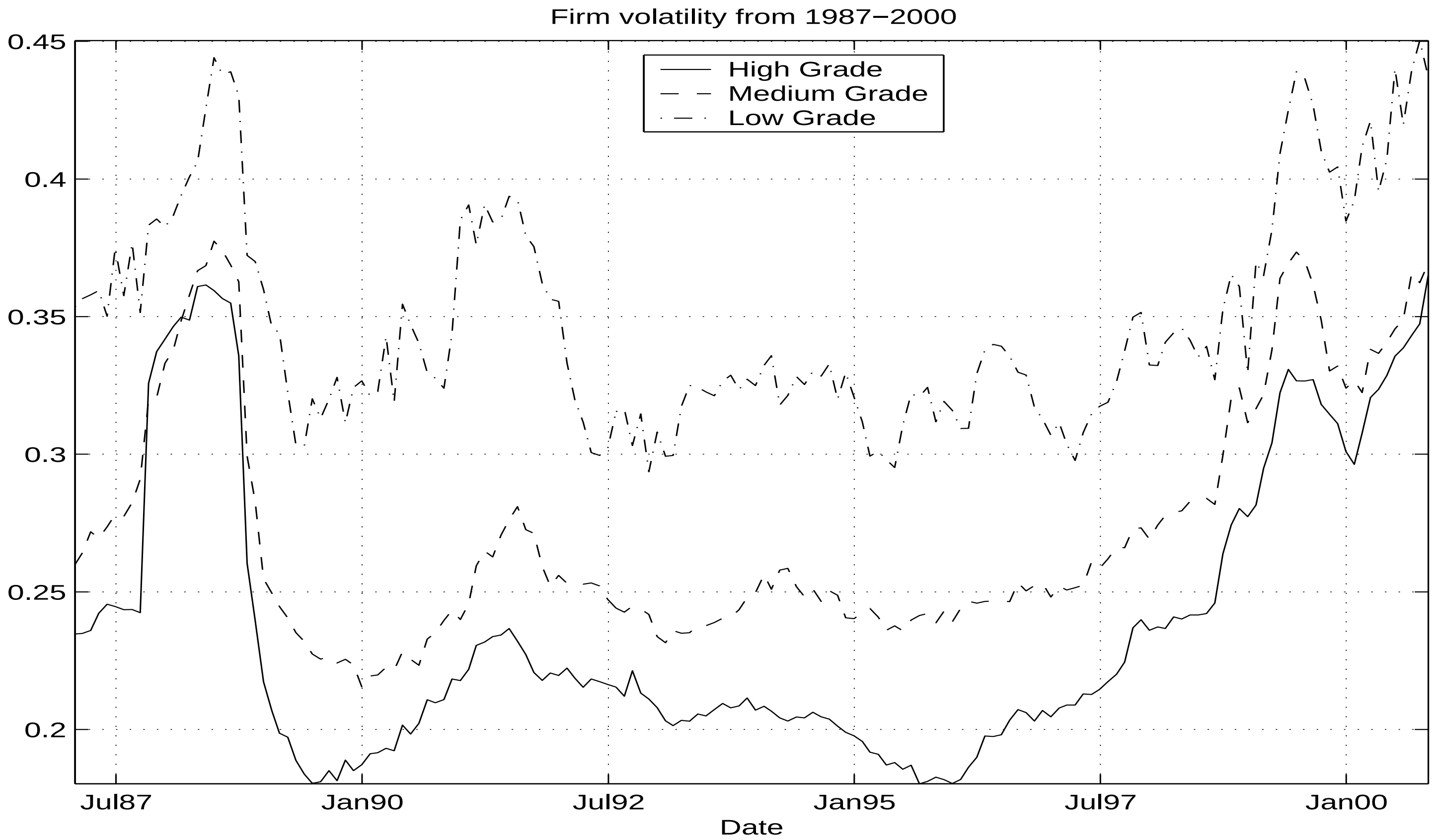
Why do corporate defaults cluster?

- Firms are exposed to common risk factors – this causes the conditional default probabilities to be correlated across firms.
- Defaults are "contagious" – default of one firm causes default of another firm, eg., the bankruptcy Penn Central Railway in June 1970 caused failures of other NE railroads.
- In the presence of unobserved or imperfectly measured risk variables, a default of a firm may reveal information about another firms – "learning from defaults".

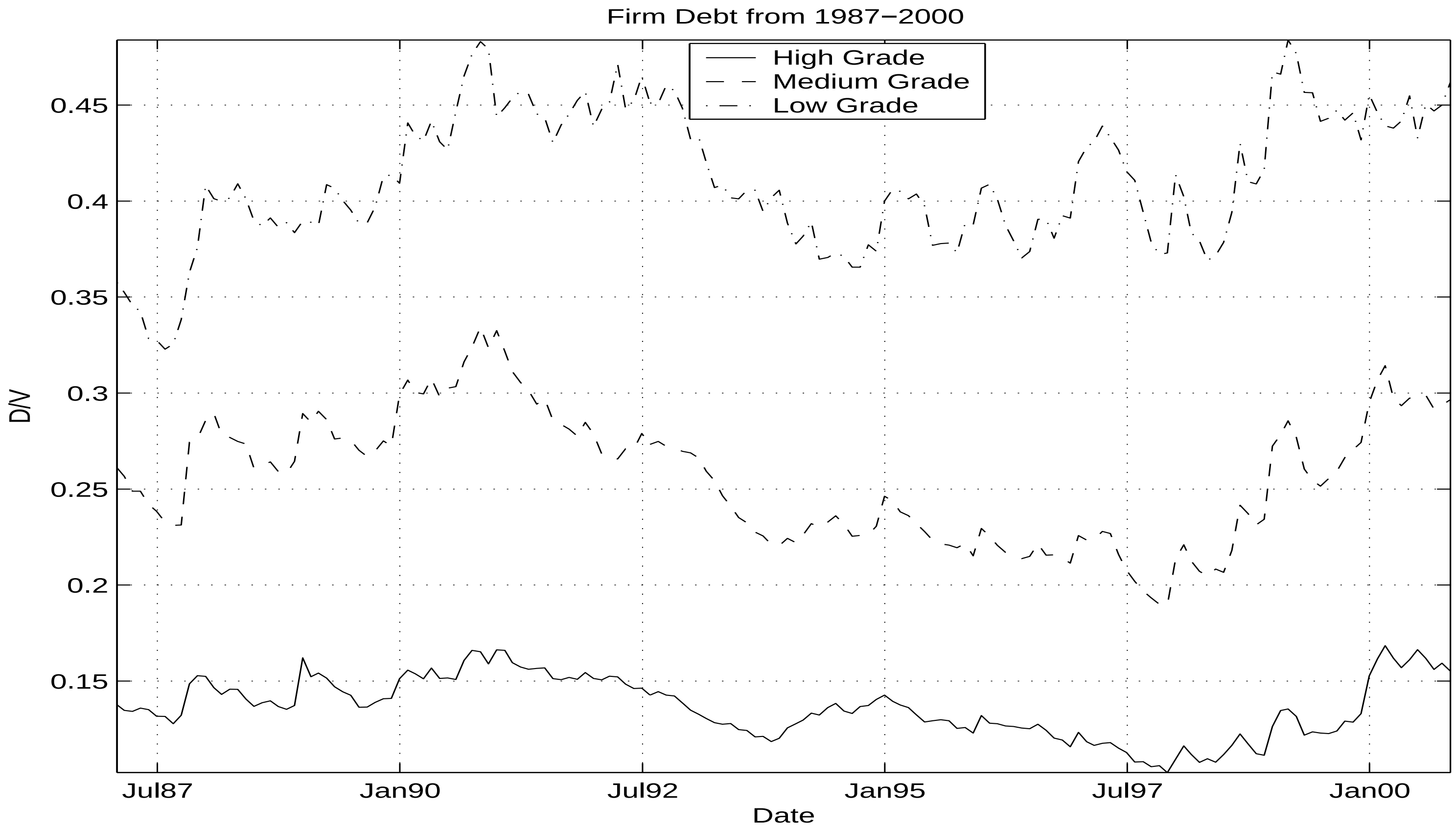
Common risk factors

- A firm's "distance to default" (DTD) is an important covariate for determining a firm's default risk. Both debt ratios and firm volatilities are correlated across firms economy-wide (Das, Freed, Geng, and Kapadia, 2001).
- Macroeconomic and economy-wide variables like GDP growth rate (Duffie and Wang, 2003), 3-month T-bill (Duffie and Saita, 2005), trailing 1-year S&P 500 return, and perhaps others (industrial production growth rate, McDonald and Van de Gucht, 1999).

Firm Volatilities (Das, Freed, Geng and Kapadia, 2002)



Firm Debt Ratios (Das, Freed, Geng and Kapadia, 2002)



Summary of Objective

- Qualitatively: Is correlation between risk factors (default intensities) sufficient to account for the degree of default clustering?
- Technically: Conditional on the path of risk factors, are defaults independent Poisson arrivals?

Why is this question important?

- Important for risk management (eg. banks' capital adequacy ratios), as well as for the pricing of securities that determine on the magnitude of default risk over a portfolio of securities (CDOs, basket default swaps).
- An entire class of credit risk models are constructed under the *doubly stochastic* assumption.
  - Conditional on the path of risk factors (intensities), defaults are independent Poisson arrivals.

- Risk management models typically assume conditional independence of default (eg. correlation between defaults comes only from correlated rating transitions, asset correlations).
- Estimates of probability of defaults (eg. Moody's KMV, Moody's Riskcalc) assumes conditional independence.

Framework: Cox Process

- Intensity may depend exogenous state variables $\mathbf{X}$.
- Default arrival by jump N_t such that $\lambda(X_t)$ is the $\mathcal{F}_t$-intensity of N.
- Also known a "doubly stochastic" process.
  1. Stochastic intensity, λ.
  2. Poisson arrival conditional on intensity.

Testing the doubly stochastic assumption

- Doubly stochastic assumption $\implies$ defaults are Poisson after conditioning on λ_t.
- Joint test of correctly specified intensities and the doubly stochastic assumption.

Findings

1. Using Moody's *Riskcalc* data on default probabilities for U.S. companies from 1987-2000, the doubly stochastic assumption is *mildly* violated.

2. But evidence of mis-specification of default intensities because of omission of economy-wide covariates.

3. Breaking news: We re-do analysis using default intensities (Duffie-Saito, 2005) that include economy-wide covariates.

4. No default clustering beyond that predicted by doubly stochastic model.

Related Literature

1. Contagion
   - Lang and Stulz (1992) - default contagion in equity prices.
2. Correlation in Process 1 -
   - Das, Freed, Geng and Kapadia (2001) - driven by market volatility, regime dependence (macro clustering).
   - Lopez (2002) and Renault and deServigny (2002) - default correlation using rating transitions.
   - Duffie & Wang (2003) - importance of macroeconomic factors.
3. Failure of the doubly stochastic hypothesis - learning from default
   - Collin-Dufresne, Goldstein and Helwege (2003) - defaults associated with spread increases, may come from (a) updated intensities or (b)increased default premia (Kusuoka 1999).
   - Jarrow & Yu (2001), Giesecke (2002), Schönbucher (2004).
   - Duffie-Lando (2000), Yu (2004) - reduction in measured precision of accounting variables results in widening of spread.
4. Clustering and copulas: Das and Geng (2004).

Data on Probabilities of Default (PDs)

- Monthly time series of one-year default probabilities for non-financial firms from Moodys *RiskCalc*, 1987-2000
- PDs based on logit model on firm-specific financial covariates and default times. Distance to default (Merton 1974) is the key co-variate; also includes balance-sheet information and rating, but not economy-wide variables.
- Calibrated to actual defaults, and comprehensive coverage (1990 firms, most firms rated by Moodys in this period).

Data on Defaults

- Default database - over 900 distinct defaults, and after removing firms without PDs, and unrated firm defaults, we have 241 distinct defaults.
- Defaults per month: average 1.5, maximum = 8, minimum = 0.
- Total default across all months = Total intensity across all months.

From PDs to Intensities

1. Default intensity process:

$$d\lambda_t = k(\theta - \lambda_t)\, dt + \sigma\sqrt{\lambda_t}\, dz_t, \tag{1}$$

2. T-maturity survival probability

$$s_t(T) = E\left[\exp\left(-\int_t^{t+T} \lambda_u\, du\right) \,\middle|\, \lambda_t\right]. \tag{2}$$

3. Cox, Ingersoll, and Ross (1985) solution:

$$
\begin{aligned}
s_t(T) &= A(T)\exp\left[-\lambda_t B(T)\right], && (3)\\
A(T) &= \left(\frac{2\gamma e^{(k+\gamma)T/2}}{(k+\gamma)(e^{\gamma T}-1)+2\gamma}\right)^{\frac{2k\theta}{\sigma^2}} && (4)\\
B(T) &= \frac{2e^{\gamma T}-1}{(k+\gamma)(e^{\gamma T}-1)+2\gamma} && (5)\\
\gamma &= \sqrt{k^2+2\sigma^2}. && (6)
\end{aligned}
$$

4. Inverting equation (3), for time horizon T,

$$
\lambda_t = -\frac{1}{B(T)}\ \ln\left[\frac{s_t(T)}{A(T)}\right]. \qquad (7)
$$

Estimation Algorithm

1. Starting coefficient estimates for $h = 1/12$,

$$s_{t+h}(1) - s_t(1) = \alpha + \beta s_t(1) + e_t, \tag{8}$$

$$k = -\frac{\beta}{h} \tag{9}$$

$$\theta = -\frac{\alpha}{\beta} \tag{10}$$

$$\sigma = \frac{V(e)}{\sqrt{\theta h}}, \tag{11}$$

   $V(e_t)$ is sample standard deviation of residual e_t.

2. Obtain initial estimate of the default intensity λ_t

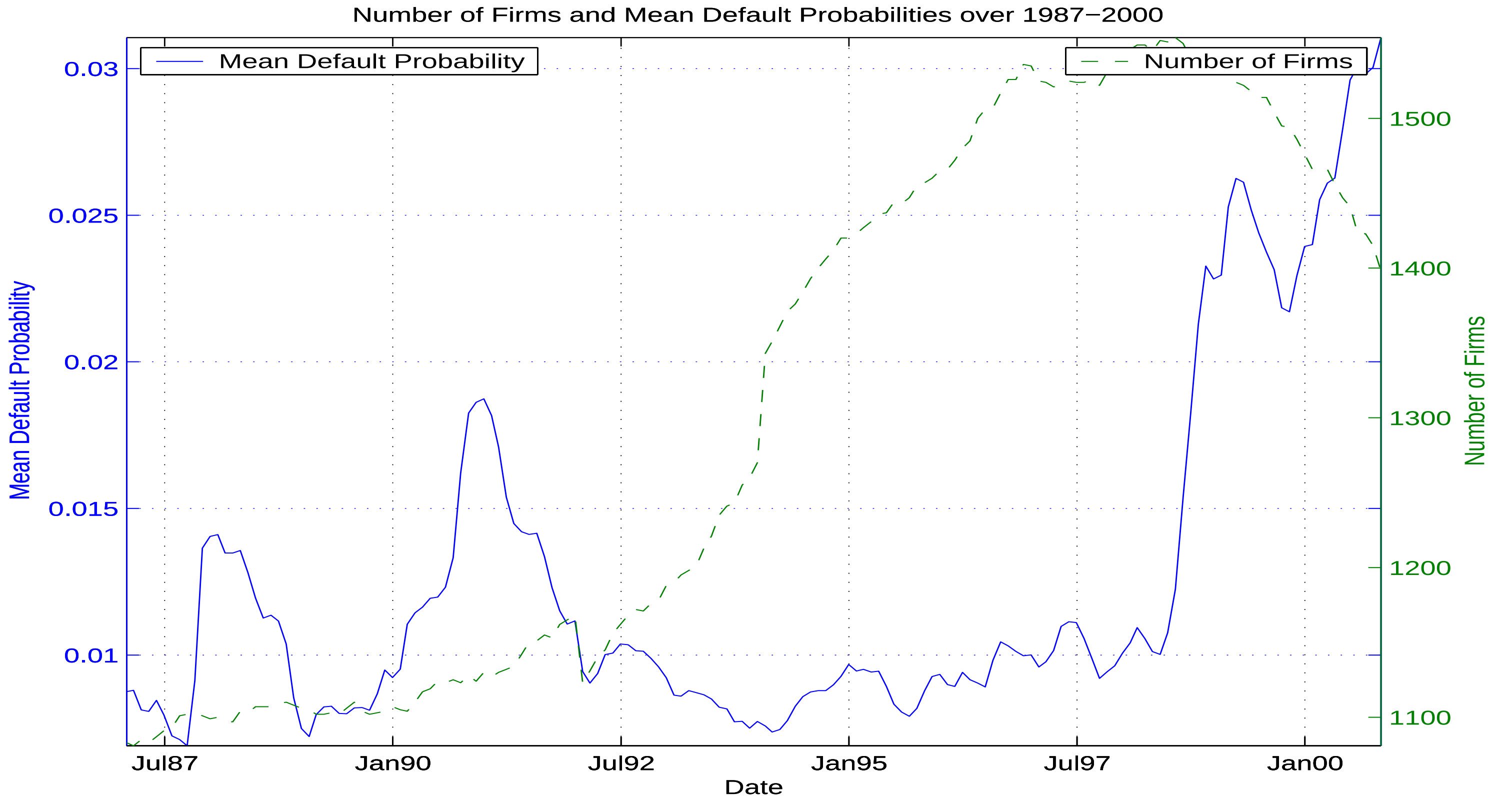
3. Next, estimate by OLS,

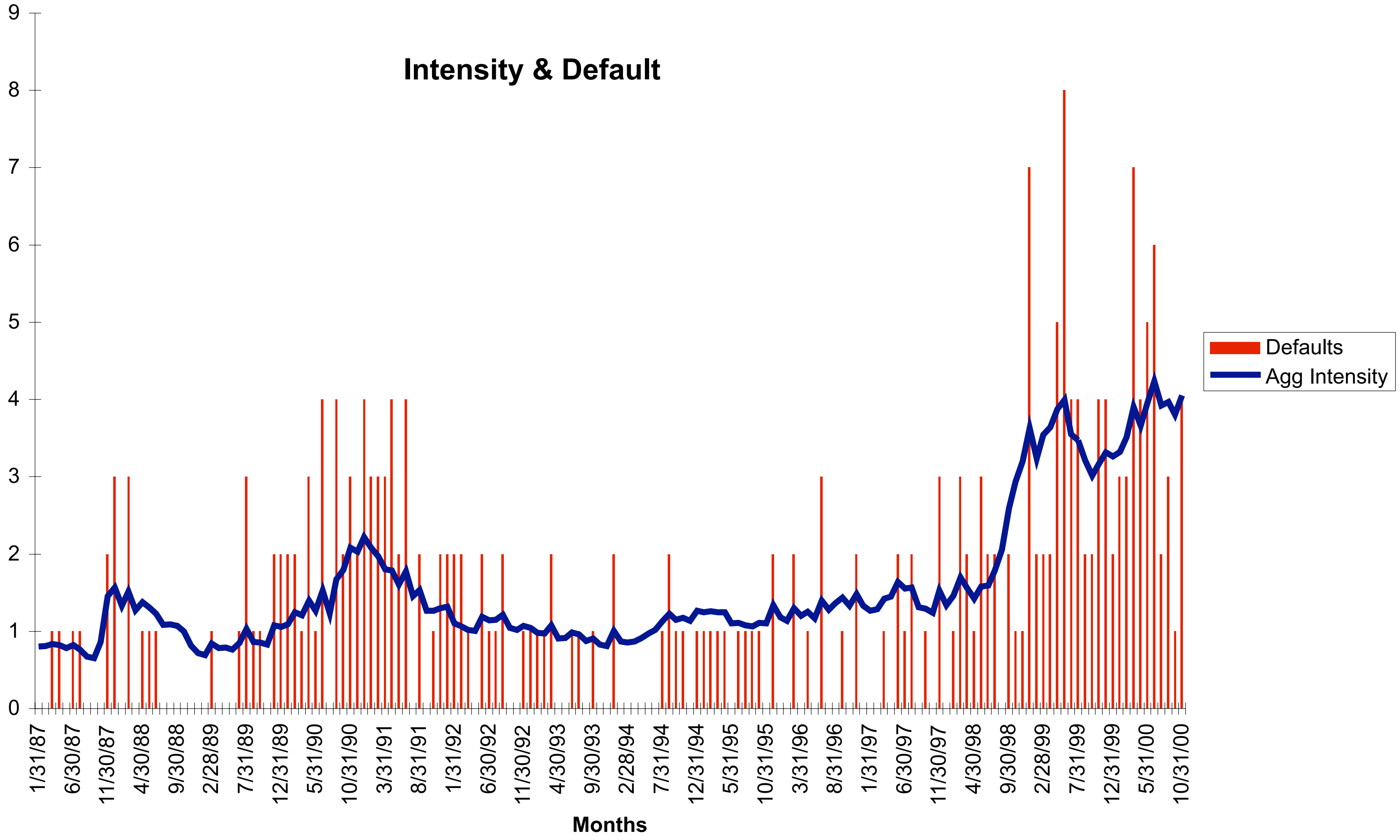
$$\lambda_{t+h} - \lambda_t = a + b\lambda_t + w_t. \tag{12}$$

New parameter estimates are then given by

$$\hat{k} = -\frac{b}{h}, \quad \hat{\theta} = -\frac{a}{b}, \quad \hat{\sigma} = V\left(\frac{w_t}{\sqrt{h\lambda_t}}\right), \tag{13}$$

4. Given these updated estimates of the parameters $\{k, \theta, \sigma\}$, iterate Steps 2 and 3, until numerical convergence.


Number of Firms and Mean Default Probabilities over 1987−2000
Mean Default Probability
Number of Firms
0.03
0.025
0.02
0.015
0.01
1500
1400
1300
1200
1100
Jul87
Jan90
Jul92
Jan95
Jul97
Jan00
Date


Intensity & Default
9
8
7
6
5
4
3
2
1
0
1/31/87
6/30/87
11/30/87
4/30/88
9/30/88
2/28/89
7/31/89
12/31/89
5/31/90
10/31/90
3/31/91
8/31/91
1/31/92
6/30/92
11/30/92
4/30/93
9/30/93
2/28/94
7/31/94
12/31/94
5/31/95
10/31/95
3/31/96
8/31/96
1/31/97
6/30/97
11/30/97
4/30/98
9/30/98
2/28/99
7/31/99
12/31/99
5/31/00
10/31/00
Months
Defaults
Agg Intensity

Proposition.
Suppose that $(\tau_1, \ldots, \tau_n)$ is doubly stochastic with intensity $(\lambda_1, \ldots, \lambda_n)$. Let $K(t) = \#\{i : \tau_i \leq t\}$ be the cumulative number of defaults by t, and let $U(t) = \int_0^t \sum_{i=1}^n \lambda_i(u) 1_{\{\tau_i > u\}}\, du$ be the cumulative aggregate intensity of surviving firms, to time t. Then $J = \{J(s) = K(U^{-1}(s)) : s \geq 0\}$ is a Poisson process with rate parameter 1.

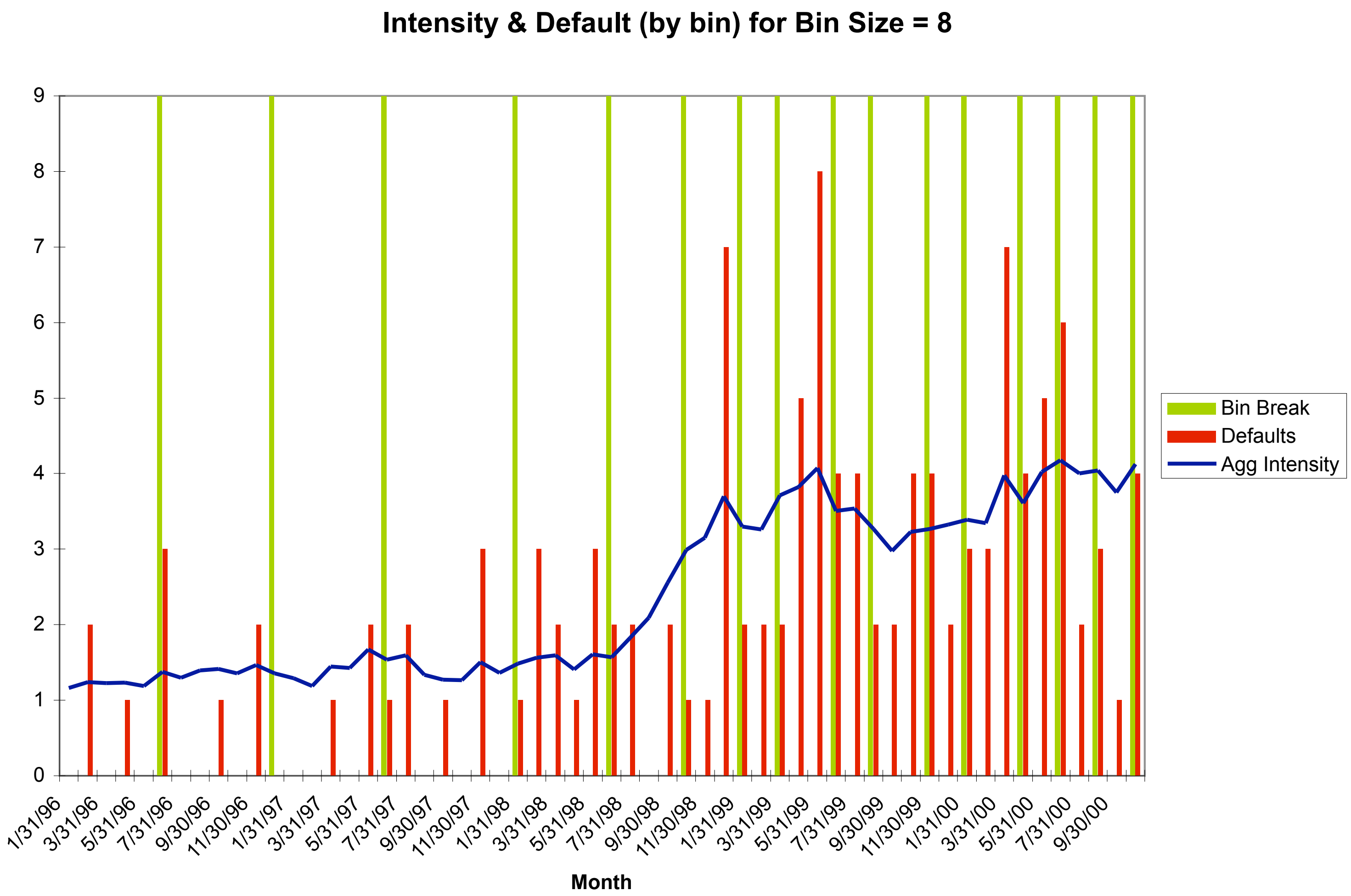
Poisson property:
For any $c > 0$, the random variables

$$J(c), J(2c) - J(c), J(3c) - J(2c), \ldots$$

are *iid* Poisson with parameter c.

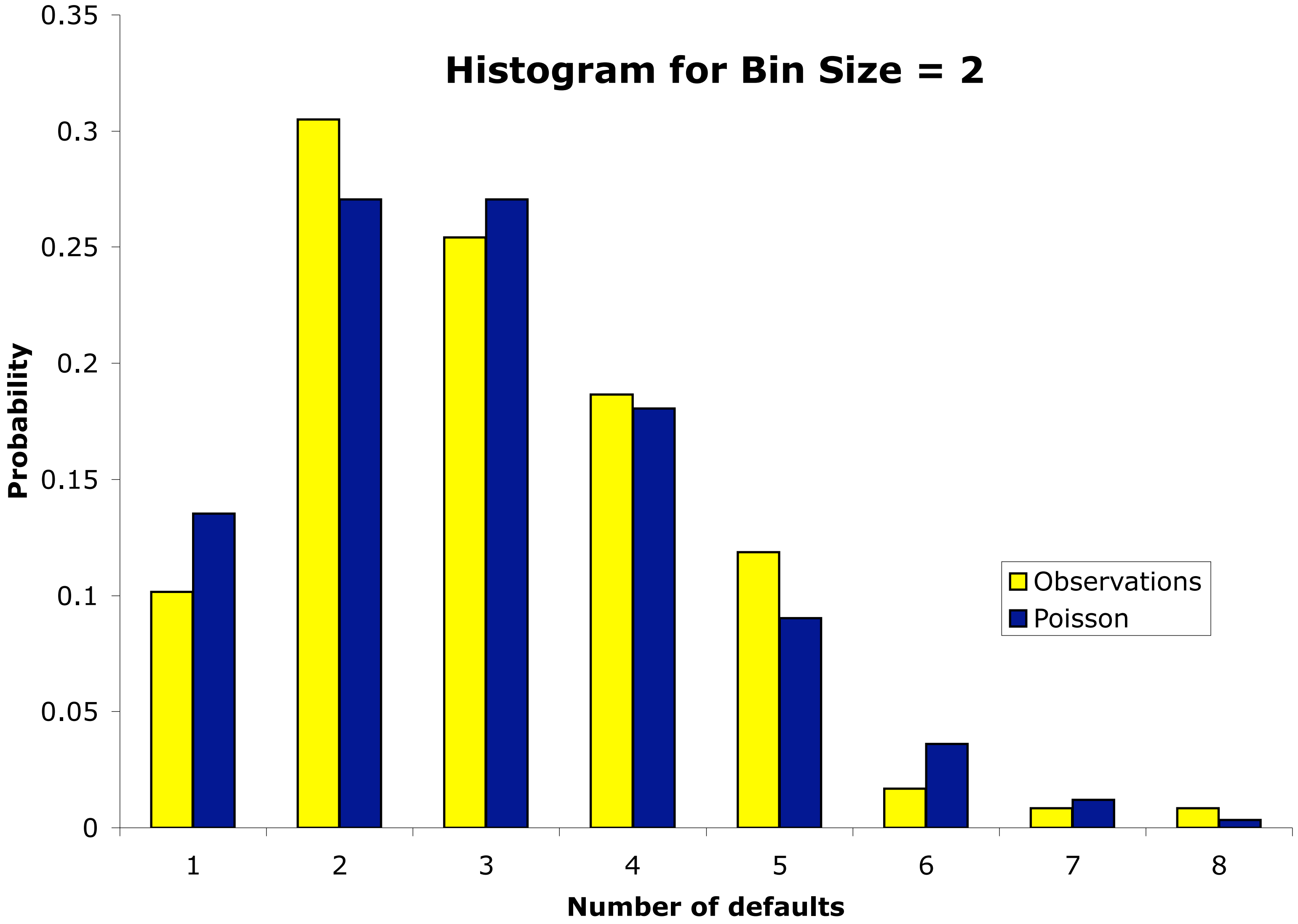
We divide our sample period into "bins" that each have an equal cumulative aggregate intensity of c, then test whether the numbers of defaults in successive bins are independent Poisson with common parameter c.

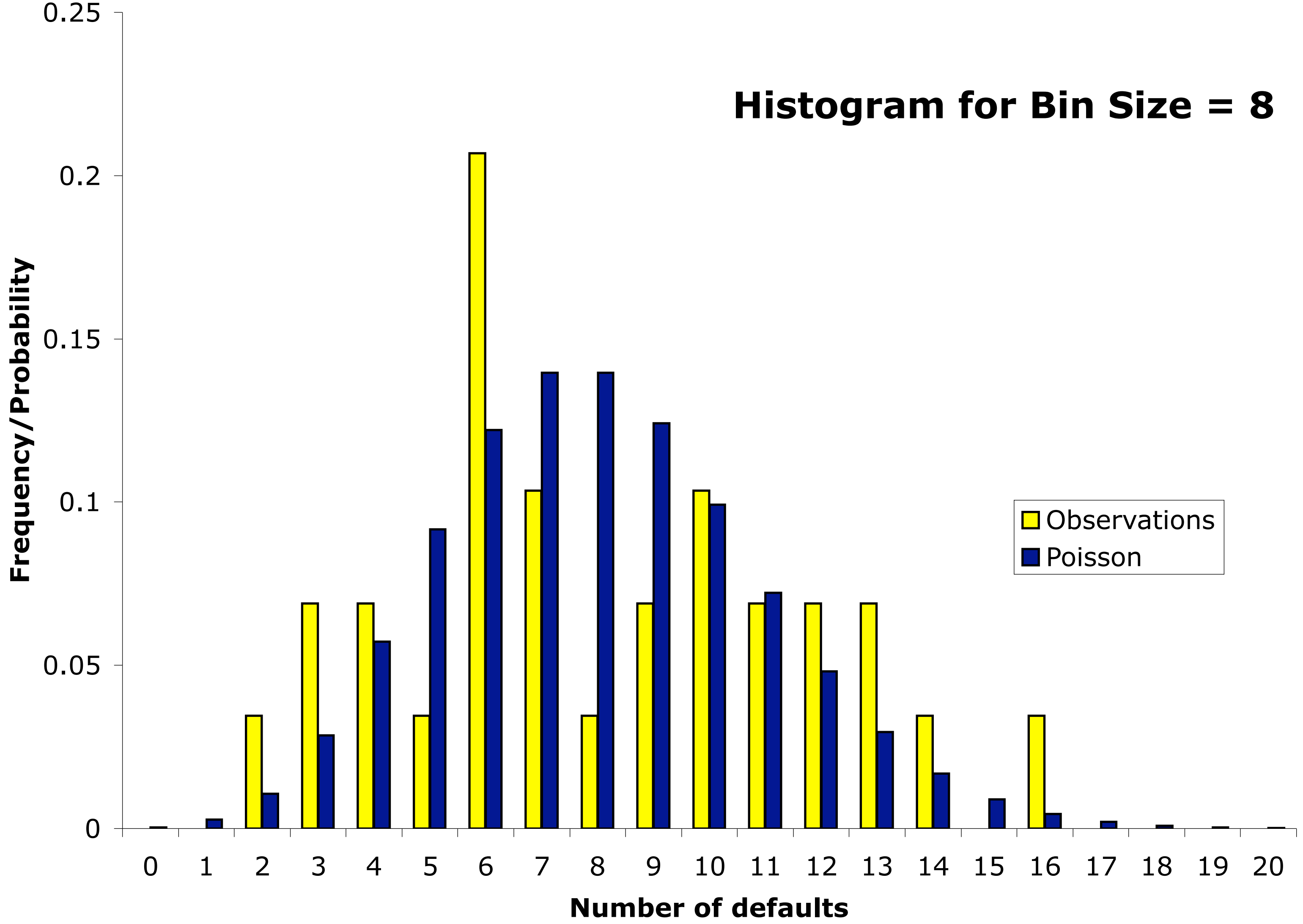
Rescaling to Intensity Time



Moments

Bin Size	Mean	Variance	Skewness	Kurtosis
2	2.00	2.00	0.71	3.50
(118)	2.04	1.89	0.71	3.52
4	4.00	4.00	0.50	3.25
(59)	4.07	4.00	0.41	2.06
6	6.00	6.00	0.41	3.17
(39)	6.08	8.07	0.41	2.19
8	8.00	8.00	0.35	3.12
(29)	8.14	13.12	0.26	2.07
10	10.00	10.00	0.32	3.10
(24)	10.04	15.43	0.82	2.25


Histogram for Bin Size = 2
Probability
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
Number of defaults
Observations
Poisson


Histogram for Bin Size = 8
Frequency/Probability
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Number of defaults
Observations
Poisson

Empirical Tests

- Standard tests for the Poisson assumption (Fisher's Chi-Sq Test, Kolmogorov-Smirnov goodness of fit)
- Upper tail tests
- Prahl's test for clustering
- Calibration of the residual (Gaussian copula) correlation.
- Tests for missing co-variates (mis-specification of the intensity process)

Fisher's Dispersion Test

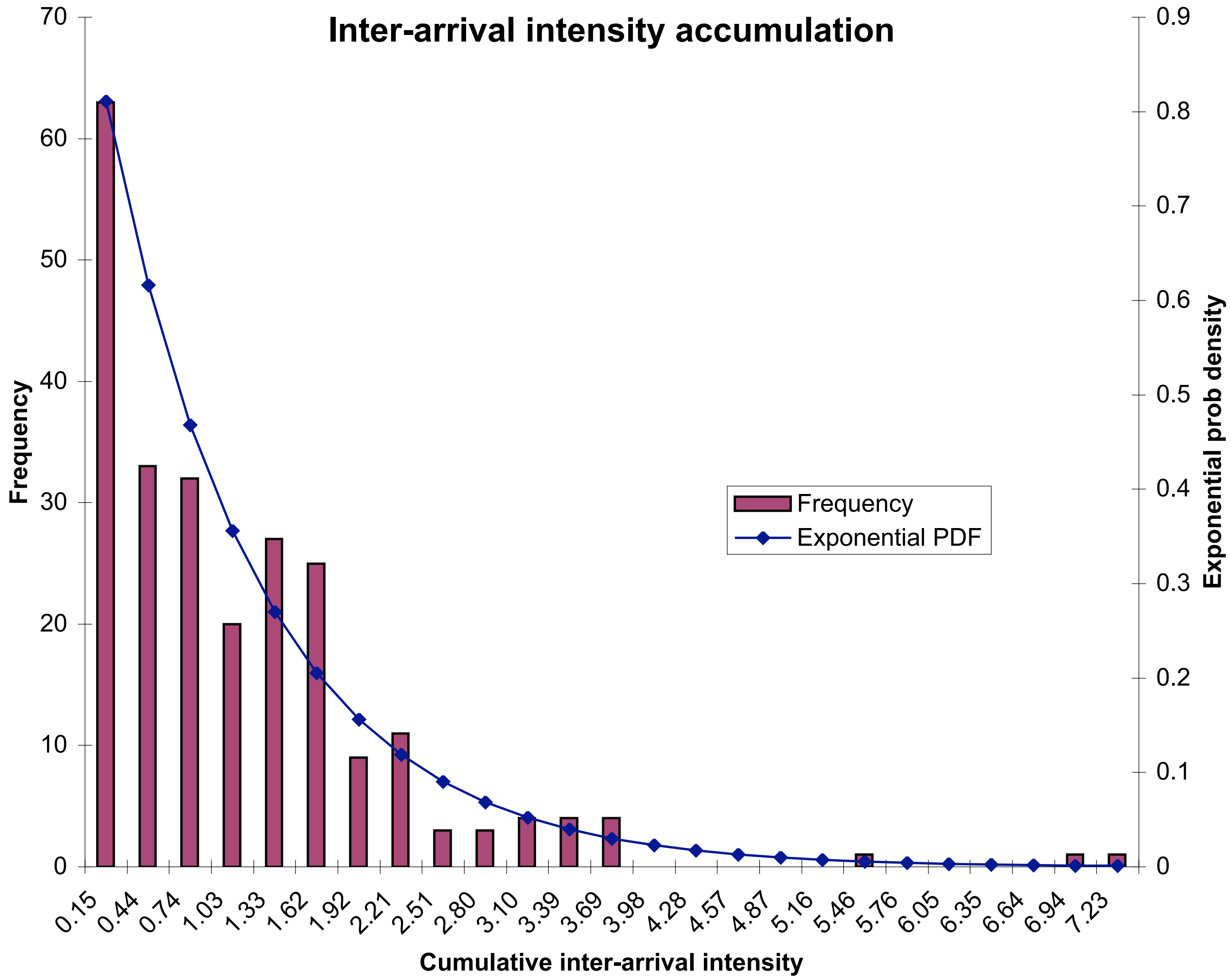
Fixing the bin size c, under the null,

$$W = \sum_{i=1}^{K} \frac{(X_i - c)^2}{c}, \tag{14}$$

is χ^2 with $K - 1$ degrees of freedom.

Bin Size	K	W	p-value
2	118	110.5	0.65
4	59	58.0	0.47
6	39	51.2	0.07
8	29	46.0	0.02
10	24	35.5	0.05

Result: borderline rejection of this null hypothesis for bin sizes 6 and 10.


Inter-arrival intensity accumulation
Frequency
Exponential prob density
Cumulative inter-arrival intensity
Frequency
Exponential PDF
70
60
50
40
30
20
10
0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.15
0.44
0.74
1.03
1.33
1.62
1.92
2.21
2.51
2.80
3.10
3.39
3.69
3.98
4.28
4.57
4.87
5.16
5.46
5.76
6.05
6.35
6.64
6.94
7.23

Examination of Interarrival Intensity

Moment	Intensity time	Calendar time	Exponential
Mean	1.07	1.07	1.07
Variance	1.19	2.19	1.16
Skewness	2.13	3.87	2.00
Kurtosis	7.46	22.01	6.00

The associated K-S statistic is 1.8681 (intensity time), (p-value = 0.0020). (For calendar time, we get K-S = 2.0716 (p-value = 0.0004), as we would expect).

Upper Tail Tests

For a given bin size c, suppose there are K bins. We let M denote the sample mean of the upper quartile of the empirical distribution of distribution of $X_1, \ldots, X_K$. By Monte Carlo simulation, we generated 10,000 data sets, each consisting of K iid Poisson random variables with parameter c. We then compute the fraction p of the simulated data sets whose sample upper-quartile size (mean or median) is above the actual sample mean M. Under the null hypothesis that the distribution of the actual sample is Poisson with parameter c, the p-value would be approximately 0.5.

Bin Size	Mean of Tails Data	Mean of Tails Simulation	p-value	Median of Tails Data	Median of Tails Simulation	p-value
2	3.62	3.63	0.58	3.00	3.18	0.25
4	6.71	6.25	0.21	6.00	5.90	0.17
6	10.00	8.81	0.05	9.50	8.42	0.07
8	12.75	11.12	0.03	12.50	10.69	0.03
10	16.00	13.71	0.02	16.50	13.26	0.00

Prahls (1999) Test of Clustered Defaults (across bin sizes)

Prahl's test statistic is based on the fact that, in the new time scale under which default arrivals are those of a Poisson process (with rate parameter 1), the inter-arrival times $Z_1, Z_2, \ldots$ are iid exponential of mean 1.

Letting C^* denote the sample mean of $Z_1, \ldots, Z_n$, Prahl shows that

$$M = \frac{1}{n} \sum_{\{Z_k < C^*\}} \left(1 - \frac{Z_k}{C^*} \right). \tag{15}$$

is asymptotically (in n) normal with mean $e^{-1} - \alpha/n$ and variance β^2/n,

$$\alpha \simeq 0.1839, \quad \beta \simeq 0.2431.$$

Empirical results in Prahl's test

1. Values under the null:

$$\begin{aligned}\mu(M) &= \frac{1}{e} - \frac{\alpha}{n} = 0.3671 \\ \sigma(M) &= \frac{\beta}{\sqrt{n}} = 0.0156.\end{aligned}$$

2. In intensity time:

   Using our data, for $n = 240$ default times, $M = 0.3681$ No notable evidence of default clustering.

3. In calendar time: $M = 0.4356$. This is evidence of a violation.

Residual Copula Correlation

- Calibrate the residual (gaussian copula) correlation (Schönbucher and Schubert, 2001). to match the excess of the upper-quartile moments of the empirical distribution of defaults per bin.
- Magnitude of calibrated Guassian copula correlation is a measure of the degree of correlation in default times that is not captured by co-movements of default intensities.

Residual Gaussian copula correlation

Bin Size	Mean of Upper quartile (data)	Mean of Simulated Upper Quartile Copula Correlation				
		$r = 0.00$	$r = 0.01$	$r = 0.02$	$r = 0.03$	$r = 0.04$
2	3.62	**3.83**	4.09	4.22	4.43	4.52
4	6.71	**6.58**	7.01	7.32	7.62	7.86
6	10.00	8.97	**9.80**	10.38	10.84	11.34
8	12.75	11.49	12.44	**12.99**	13.76	14.77
10	16.00	13.82	14.69	**15.86**	16.69	17.39

Test of Independence of Successive Defaults

Estimates of an auto-regressive model for a range of bin sizes (t-statistics are shown parenthetically). [Mild autocorrelation]

Bin Size	No. of Bins	A (t_A)	B (t_B)	R^2
2	118	1.73	0.16	0.03
		(7.66)	(1.72)	
4	59	2.72	0.34	0.12
		(4.83)	(2.73)	
6	39	4.20	0.32	0.10
		(3.97)	(2.01)	
8	29	6.68	0.19	0.03
		(3.83)	(0.96)	
10	24	6.09	0.39	0.15
		(2.75)	(1.93)	

PD Mis-specification

Fix bin size c. Defaults in a bin in excess of the mean, $Y_k = X_k - c$, GDP is GDP growth rate, IP is industrial production growth rate.

$$Y_k = \alpha + \beta_1 GDP_k + \beta_2 IP_k + +\epsilon_k, \tag{16}$$

Bin Size	No. Bins	Intercept	GDP	IP	R^2 (%)
2	118	0.55 (3.06)	-11.99 (-2.44)	-44.29 (-1.77)	8.27
4	59	0.90 (1.86)	-17.12 (-1.48)	-88.66 (-2.51)	11.69
6	39	1.43 (1.83)	-27.53 (-1.32)	-139.34 (-2.14)	14.21
8	29	1.71 (1.43)	-21.02 (-0.63)	-276.08 (-2.64)	18.99
10	24	3.83 (4.01)	-64.67 (-2.36)	-360.47 (-2.59)	39.98

Upper-tail regressions.
Regression of the number of defaults observed in the upper quartile less the mean of the upper quartile of the theoretical distribution (with Poisson parameter equal to the bin size).

Bin Size	K	Intercept	Previous Qtr GDP	Previous Month IP	R^2 (%)
2	40	-0.07	8.56	-75.57	20.98
		(-0.55)	(1.99)	(-2.79)	
4	17	0.66	-7.13	-18.57	10.76
		(2.37)	(-1.15)	(-0.57)	

Bin Size	K	Intercept	Current Bin GDP	Current Bin IP	R^2 (%)
2	40	-0.16	-10.32	-57.99	8.24
		(-0.95)	(1.19)	(-1.58)	
4	17	0.43	3.82	-35.63	10.90
		(1.96)	(0.45)	(-1.92)	

Summarizing Results from Moody's RiskCalc Default Probabilities

1. We reject the joint hypothesis of correctly measured intensities and the the doubly stochastic assumption.
2. There is only mild evidence that defaults are more tightly clustered in time than that implied by their intensities.
3. There is evidence of PD mis-specification because of omission of macroeconomic covariates.

Breaking News

1. If, indeed, the rejection of the Poisson assumption is driven by mis-specification, then we should observe no (or even milder) rejection if we use PDs that account for systematic factors .
2. Duffie and Saita (2005) estimate PDs and find the Tbill rate, 1-yr trailing SPX return, cross-sectional industry average DTD as significant co-variates. (After accounting for these, GDP, PI, IP growth rates are not significant.)
3. When we use this data, we do not reject the doubly stochastic assumption towards higher clustering and

find no mis-specification.

4. Specifically, we do not reject that the upper-tail is consistent with the Poisson assumption, we do not reject using Prahl's test, the residual Gaussian copula correlation is < 0.01, we do not find autocorrelation across bins, and we do not find that excess defaults are correlated with macroeconomic variables.

Moments (214 total defaults)

Bin Size	Mean	Variance	Skewness	Kurtosis
2	2.03	2.03	0.70	3.49
(116)	1.83	2.16	0.99	3.97
4	4.03	4.03	0.50	3.25
(58)	3.66	5.07	0.99	4.16
6	6.02	6.02	0.41	3.17
(39)	5.44	6.94	0.68	3.40
8	8.03	8.03	0.35	3.12
(29)	7.28	12.06	0.32	2.73
10	10.03	10.03	0.32	3.10
(23)	9.13	15.12	0.11	2.34

Fisher's Dispersion Test

Fixing bin size c, under the null,

$$W = \sum_{i=1}^{K} \frac{(X_i - c)^2}{c}, \tag{17}$$

is χ^2 with $K - 1$ degrees of freedom. Result: rejection of null for bins 8, 10.

Bin Size	K	W	p-value
2	116	126.0	0.23
4	58	74.0	0.06
6	39	46.0	0.17
8	29	44.1	0.03
10	23	35.0	0.04

Upper Tail Tests

We cannot reject that mean of upper tail is consistent with Poisson assumption.

Bin	Mean of Tails		p-value	Median of Tails		p-value
Size	Data	Simulation		Data	Simulation	
2	3.90	3.64	0.08	4.00	3.19	0.00
4	6.15	6.24	0.48	5.50	5.90	0.73
6	8.38	8.81	0.70	8.00	8.41	0.50
8	11.71	11.39	0.35	11.00	10.96	0.33
10	14.00	13.63	0.34	14.00	13.17	0.17

Empirical results in Prahl's test

1. Using our data, for $n = 213$ default times, $M = 0.3887$
2. No evidence of default clustering (t-value=1.31).

Residual Gaussian copula correlation < 0.01

Table 5: Residual Gaussian copula correlation.

Bin Size	Mean of Upper quartile (data)	Mean of Simulated Upper Quartile Copula Correlation				
		$r = 0.00$	$r = 0.01$	$r = 0.02$	$r = 0.03$	$r = 0.04$
2	3.90	**3.84**	4.00	4.18	4.30	4.41
4	6.15	**6.50**	6.84	7.10	7.42	7.75
6	8.38	**8.83**	9.47	9.77	10.33	10.72
8	11.71	11.20	**11.89**	12.40	13.07	13.71
10	14.00	13.44	**14.24**	14.98	15.79	16.56

Test of Independence of Successive Defaults Estimates of an auto-regressive model for a range of bin sizes
(t-statistics are shown parenthetically). No autocorrelation across bins.

Bin Size	No. of Bins	A (t_A)	B (t_B)	R^2
2	116	1.6951	0.0714	0.005
		(7.67)	(0.76)	
4	58	3.4350	0.0629	0.004
		(5.89)	(0.47)	
6	39	4.9996	0.0718	0.005
		(4.91)	(0.43)	
8	29	8.2518	-0.1349	0.017
		(5.06)	(-0.67)	
10	23	9.7970	-0.0761	0.006
		(4.32)	(-0.34)	

Tests for PD Mis-specification: Overall, no evidence of mis-specification on account of missing economy-wide covariates

Defaults in a bin in excess of the mean, $Y_k = X_k - c$, GDP is GDP growth rate, IP is industrial production growth rate.

$$Y_k = \alpha + \beta_1 GDP_k + \beta_2 IP_k + +\epsilon_k, \tag{18}$$

Bin Size	No. Bins	Intercept	GDP	IP	R^2 (%)
2	116	-0.14 (-0.54)	3.50 (0.55)	-55.27 (-2.24)	3.54
4	58	-0.54 (-0.95)	8.34 (0.54)	-53.66 (-1.06)	1.75
6	39	-0.17 (-0.20)	-9.00 (-0.44)	-66.58 (-.95)	3.55
8	29	-1.52 (-1.17)	32.09 (0.82)	-135.79 (-0.77)	4.47
10	23	0.48 (0.25)	-32.81 (-0.65)	-101.58 (-0.48)	5.47

Regression of the number of defaults observed in the upper quartile less the mean of the upper quartile of the theoretical distribution (with Poisson parameter equal to the bin size).

Bin Size	K	Intercept	Previous Qtr GDP	Previous Month IP	R^2 (%)
2	29	0.61 (1.15)	-10.33 (-0.91)	8.79 (0.22)	5.05
4	20	0.22 (0.35)	-11.31 (-0.65)	84.33 (1.12)	6.76

Bin Size	K	Intercept	Current Bin GDP	Current Bin IP	R^2 (%)
2	29	0.13 (0.68)	-3.70 (-0.55)	55.76 (1.21)	6.67
4	20	0.07 (0.15)	26.50 (0.95)	-104.20 (-1.73)	21.72

Conclusion

1. Overall, we do not reject the DS assumption.
2. The clustering of defaults in time can be explained by correlations between default intensities, and there appears to be no need to model any additional residual correlation.
3. It appears necessary, however, that economy-wide covariates are included in the estimation of the default intensity process.