The Valuation of Correlation-Dependent Credit Derivatives Using a Structural Model

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Motivation for Paper

"Banks insurance companies and other financial institutions managing portfolios of credit risk need an integrated model, one that reflects correlations in default and changes in market spread. Yet no such model exists."

Darrell Duffie, Financial Times, April 16, 2004

Background

- Merton (1974)
- Black and Cox (1976)
- Zhou (2001)
- Hull and White (2001)
- Gregory and Laurent (2003)
- Hull, Predescu and White (2005)

Gaussian Copula Model

- Has become market standard for modeling default correlation
- One (or more) factors drives correlation between times to default of different obligors
- Easy to implement
- One-to-one correspondence between market prices and correlation parameter

Gaussian Copula Model: Disadvantages

- The realization of a single factor governs the default environment in all future time periods
- No model for evolution of credit spreads or correlations
- No underlying economic rationale
- No way of knowing what the copula correlations should be

A Factor-Based Structural Model

$$dV_i(t) = \mu_i V_i(t) + \sigma_i V_i(t) dX_i(t)$$

where $V_i(t)$ is the value of the assets of company i at time t

There is a barrier H_i such that the company defaults when V_i falls below H_i for the first time

The barrier for X_i is $\beta_\text{i} + \gamma_i t$ where

$$\beta_i = \frac{\ln H_i - \ln V_i(0)}{\sigma_i} \qquad \gamma_i = -\frac{\mu_i - \sigma_i^2/2}{\sigma_i}$$

The probability of first hitting this barrier between times t and $t+T$ is

$$N\left(\frac{\beta_\text{i} + \gamma_i(t+T) - X_i(t)}{\sqrt{T}}\right) + \exp[2(X_i(t) - \beta_i - \gamma_i t)\gamma_i] N\left(\frac{\beta_i + \gamma_i(t-T) - X_i(t)}{\sqrt{T}}\right)$$

The Correlation Model

$$dX_i(t) = \alpha_i(t)dF(t) + \sqrt{1-\alpha_i(t)^2}\,dU_i$$

where F is a common factor affecting all asset prices and the U_i are uncorrelated with each other and uncorrelated with F

Fitting the Model to Market Data on CDX and iTraxx

- We choose β and γ (same for all companies) to match the 5- and 10-year indices
- On August 24, 2004 these indices were 59.73 and 81.00 for CDX
- This leads to β=-3.89 and γ=-0.12 and means that

$$\frac{H}{V(0)} = 0.5583 \qquad \mu = 0.0289$$

CDS Spreads in bps (MDD is number of SDs by which asset price exceeds barrier)

MDD	Horizon				
	1	3	5	7	10
2.50	55.22	226.61	251.64	246.32	229.98
3.00	11.28	115.74	153.01	160.66	157.41
3.50	1.83	55.79	91.04	104.14	108.31
4.00	0.23	25.16	52.54	66.51	74.29
4.50	0.02	10.57	29.24	41.62	50.51
5.00	0.00	4.12	15.64	25.41	33.92

Drift of CDS spread per week (bps)

Drift of CDS Spreads per week (in bps)					
MDD	Horizon				
	1	3	5	7	10
2.50	1.1	2.0	1.7	1.4	1.2
3.00	0.2	0.9	0.9	0.8	0.6
3.50	0.0	0.4	0.5	0.4	0.4
4.00	0.0	0.2	0.2	0.3	0.2
4.50	0.0	0.1	0.1	0.2	0.1
5.00	0.0	0.0	0.1	0.1	0.1

SD of CDS spread per week (bps)

Standard Deviation of CDS Spreads per week (in bps)					
MDD	Horizon				
	1	3	5	7	10
2.50	22.6	40.7	34.3	29.3	24.6
3.00	5.3	22.4	21.5	19.1	16.4
3.50	1.0	11.8	13.5	12.7	11.2
4.00	0.1	5.8	8.3	8.4	7.8
4.50	0.0	2.7	4.9	5.5	5.5
5.00	0.0	1.1	2.8	3.6	3.8

Comparison with Gaussian copula model

- We find that our base-case structural model with a particular correlation parameter gives very similar joint probabilities and CDO tranche prices to the basic Gaussian copula model with the same correlation parameter
- This is not too surprising because the two models are the same if we assume that once an asset price has dipped below its barrier it stays below the barrier
- However this does not mean that extensions to the Gaussian copula model are economically reasonable

Typical Daily Data

	CDX IG Tranches					
	0% to 3%	3% to 7%	7% to 10%	10% to 15%	15% to 30%	0% to 100%
5-year Quotes	40.02%	295.71	120.50	43.00	12.43	59.73
10-year Quotes	58.17%	632.00	301.00	154.00	49.50	81.00

	iTraxx IG Tranches					
	0% to 3%	3% to 6%	6% to 9%	9% to 12%	12% to 22%	0% to 100%
5-year Quotes	24.10%	127.50	54.00	32.50	18.00	37.79
10-year Quotes	43.80%	350.17	167.17	97.67	54.33	51.25

Fitting the Correlation Parameter

- We carried out two sets of tests
  - We assumed a constant correlation parameter, fitted it to the 5-year equity tranche, and observed the pricing of other tranches
  - We assumed a step function for the correlation parameter, fitted it to the 5 and 10 year equity tranches, and observed the pricing of other tranches.

Results

- The basic structural model fits market quite poorly. The mezzanine tranche is overpriced and the super-senior tranche is underpriced
- Extending the structural model so that the recovery rate is positively correlated with F makes very little difference
- Extending the structural model so that the correlation parameter is negatively correlated with F fits market data much better.

Conclusions

- Structural model provides a way of jointly modeling default events and credit spreads.
- A positive correlation between default correlation and default rates may explain CDO pricing observed in the market