

The Valuation of Correlation-Dependent Credit Derivatives Using a Structural Model

Discussion

Second Credit Risk Conference

May 26, 2005

Gifford Fong

GIFFORD FONG ASSOCIATES

3658 Mount Diablo Boulevard, Suite 200

Lafayette, CA 94549

Ph.: (925) 299-7800 Fax: (925)299-7815

Email: gfa@gfong.com

www.gfong.com

Background on Default Correlation

- Default correlation exists and changes with the economic situation (e.g. Das, Freed, Geng and Kapadia [2004], Lucas [1995])
- Modeling default correlations is needed in both valuation and portfolio management.
- Three ways of modeling default correlations:
 - Latent variable approach based on the structural model in which asset correlations are used. (e.g. Zhou [2001],Hull and White [2001])
 - Intensity models where default correlations are obtained through the modeling of intensity correlations. (e.g. Davis and Lo [2001], Duffie and Singleton [1999], and Jarrow and Yu [2001])
 - Copula survival time models where the copula correlations are defined. (e.g. Laurent and Gregory [2003],Li [2000])

Contributions of the Paper

- The proposed model is a multi-factor model of correlated default.
- Incorporates correlated default of multiple issues into a first passage time structural model.
- Incorporates stochastic recovery.
- Analyzes the model and compares with the survival time Copula model.
- Prices correlation-dependent derivatives (CDO tranches and options on CDO tranches).

Advantages of Proposed Model

- Asset correlations are used and determined by one or more factors.
- Economic rationale for the model.
- Credit spreads are time-dependent, which can be applied to very complicated structures involving both correlated default and correlated spread uncertainty.
- Correlations evolve through time and are dependent on default rates.
- Easy to implement and computationally fast.

Model Framework

The assets of firm i is modeled by the following process

$$dV_i = \mu_i V_i dt + \sigma_i V_i dX$$

or

$$d \ln V_i = (\mu_i - \sigma_i^2/2) dt + \sigma_i dX_i$$

The company defaults for the first time at time t when

$$X(t) \leq H(t)^*$$

Where

$$X(t) = \frac{\ln V_i(t) - \ln V_i(0) - (\mu_i - \sigma_i^2/2)t}{\sigma_i}, X_i(0) = 0$$
$$H_i^* = \frac{\ln H_i - \ln V_i(0) - (\mu_i - \sigma_i^2/2)t}{\sigma_i}$$

and H_i is a barrier.

Assume

$$dX_i(t) = \sum_{j=1}^{n_F} \alpha_{ij}(t) dF_j(t) + \sqrt{1 - \sum_{j=1}^{n_F} \alpha_{ij}(t)^2} dU_i(t) \quad (1)$$

Then,

$$Cor(dX_{i_1}, dX_{i_2}) = \sum_{j=1}^{n_F} \alpha_{i_1 j} \alpha_{i_2 j}$$

Implementation

For each company,

Bond prices or CDS \rightarrow Risk-neutral PD at time $t \rightarrow$ Default barrier $H_i(t)$.

For the portfolio,

Simulate factors $F(t)$ and $U_i(t) \rightarrow$ Default event for Each company by equation (1) \rightarrow Present value of payoff of the credit derivative under the recovery assumption.

Multi-step Versus Single-step Approach

- The relationship between the framework of the structural model and the survival time Copula model is similar to multi-step versus single-step approaches
- As a multi-step approach, the structural model has a time varying capability with respect to all its parameters. This leads to
 - The factors change from period to period. As a result, the asset value and the default outcome change accordingly.
 - The credit spread of a company, which is a function of asset prices, the barrier, interest rates and the recovery rate, evolves through time.
- The factor exposure α_i is dependent on time and other elements such as default rate, resulting in time varying correlations.
- In contrast, the survival time method determines the default time for each company based on the realization of all the parameters at the beginning. This leads to:
 - Constant correlation
 - Constant credit spread
 - Simplified implementation
- The structural model allows pricing of cash CDOs whereas the default time copula only allows pricing of synthetic CDOs.

Issues and Future Directions

- How is the implementation of the copula model done for the CDX IG base case (pg 16)?
- The Das, Freed, Geng and Kapadia (2004)(DFGK) model uses Moodys PDs which are structural, not reduced form. The correlation analysis is not based on either structural or reduced form.
- Will the structural model apply to other situations other than credit index tranches, whose underlying companies are usually equal weighted and assumed to have the same term structure of default probabilities? When applied to a cash flow CDO where each underlying security has to be considered separately, is the structural model still computationally efficient?
- Although the results for the single tranches seem close for both the structural model and the survival time model, are the values close for long term and complicated structured products?

More Issues and Future Directions

- Have the underlying securities been eliminated from the portfolio upon default in the valuation of structured products especially when the default probabilities are different for different assets?
- Asset correlations are a key input to the structural model. How do we get asset correlations?
- Since the state variable X generated from the asset value process follows a Weiner process, dX can only follow a Normal distribution. This reduces the modeling flexibility compared with the Copula method. Does this affect the computational accuracy?
- Some disadvantages (such as constant spread and fixed default environment) of the copula model arise when it is used as one-period model. Actually, the copula model may also be applied over multi-periods, which may eliminate those disadvantages.

Summary

- The structural model has an economic rationale in which asset correlations can be considered.
- The factor representation of the asset value provides computational efficiency.
- The multi-step process allows time varying characteristics of credit spreads and correlations, which are valuable in pricing complicated credit derivatives.
- Although the framework is in the risk-neutral environment for valuation purposes, it can also be used in the physical world where the physical probability of default will be used for risk management.
- The computational efficiency for some complicated credit derivatives can be addressed.