A model of swap spreads and corporate bond yields

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Main goals

- Proposing and estimating a joint pricing model for swaps, corporate bonds, and treasuries

- Decomposing swap spreads into credit and liquidity components. Identifying the idiosyncratic swap factor.

- Is LIBOR-GC repo a good measure of short term AA credit risk?

- Estimating a 'convenience yield' term structure for treasuries and comparig with refcorp - treasury spreads

Most directly related literature

- Collin-Dufresne and Solnik (JF, 2001)
- Duffee (RFS, 1999), Duffie and Singleton (JF, 1997)
- Liu, Longstaff and Mandell (WP, 2003), He (WP, 2001)
- Grinblatt (IRF, 2001)
- Reinhart and Sack (2002)
- Lando (1998)

Model specification: The latent factors

State vector X consists of 6 independent diffusion processes with an affine drift and volatility structure with P and Q evolution

$$
\begin{aligned}
X_t &= (X_{1t}, ..., X_{6t})' \\
dX_{it} &= k_i(X_{it} - \theta_i)dt + \sqrt{\alpha_i + \beta_i X_{it}} dW_i^P, i = 1, ..., 6, \\
dX_{it} &= k_i^* X_{it} dt + \sqrt{\alpha_i + \beta_i X_{it}} dW_i^Q, i = 1, ..., 6,
\end{aligned}
$$

For identification purposes, we normalize the $Q-$means to be zero.

Affine technology allows to price the different securities

The riskless rate and the Treasury securities

- $r^g(X) = a + X_1 + X_2$ (The government short rate)

- $r(X) = r^g(X) + (e + X_5)$ (The riskless rate)

- $e + X_5$ is the convenience yield associated with holding treasuries.

- The price of the treasuries depends on two factors and has the form

$$P^g(t,T) = \exp(A^g(T-t) + \mathbf{B^g}(T-t)'X_t)$$

The corporate bonds

We model simultaneously the yield curves for four different rating classes in banks and financials.

The price at time t of zero-coupon bond rated i at t and maturing at T is

$$v^i(t,T) = E_t^Q \,\mathsf{exp}\left(-\int_t^T (r(X_u) + \lambda(X_u,\eta_u)du)\right)$$

The corporate bonds

The pricing formula

$$v^i(t,T) = E_t^Q \exp\left(-\int_t^T (r(X_u) + \lambda(X_u, \eta_u)du)\right)$$

requires us to specify the default intensity for each state and the migration between non-default states:

$$\begin{aligned}
\lambda(X,i) &= \nu_i \mu(X_s) \qquad \text{(loss-adjusted default rate)} \\
a_{ij}(X_t) &= \lambda_{ij}\mu(X_t) \qquad \text{(migration)} \\
\mu(X) &= b + X_3 + X_4 + c(X_1 + X_2)
\end{aligned}$$

Interpret μ as a common random factor controlling migration intensities and default-rates

The corporate bonds

$$a_{ij}(X_t) = \lambda_{ij}\mu(X_t) \qquad \text{(migration)}$$

requires the input of a baseline generator matrix

$\tilde{A}$	AAA	AA	A	BBB	SG
AAA	-0.0976	0.0847	0.0122	0.0007	0
AA	0.0157	-0.1286	0.1090	0.0028	0.0011
A	0.0010	0.0267	-0.1012	0.0678	0.0057
BBB	0.0009	0.0024	0.0669	-0.1426	0.0723
SG	0	0.0004	0.0066	0.1220	-0.1291

The 'baseline' intensities after collapsing spec grades into one category

The corporate bonds

The price of a zero coupon corporate bond in rating class i at time t is of the form:

$$v^i(t,T) = \sum_{j=1}^{K-1} c_{ij} E_t(\text{exp}(\int_t^T d_j \mu(X_u) - r(X_u) du))$$

where the constants c_{ij} and d_j can be computed explicitly

The swap rates

The short rate on the swap as set on date t and paid at date $t+0.25$ is modelled as

$$L(t, t+0.25) = \frac{1}{v^{LIB}(t, t+0.25)} - 1$$

where

$$\begin{aligned} v^{LIB}(t, t+0.25) &= E_t^Q \exp\left(-\int_t^{t+0.25} \lambda^{LIB}(X_s) ds\right) \\ \lambda^{LIB}(X_s) &= r(X_s) + \nu_{AA}\mu(X_s) + S(X_s) \\ S(X) &= d + X_6 \end{aligned}$$

$S(X) = 0$ would correspond to an assumption of homogeneous LIBOR-swap market credit quality, i.e. that the short AA corporate rate and LIBOR were the same.

The swap rates

Assume that the swap rate can be found by discounting both sides of the swap using the riskless rate.

This corresponds to ignoring counterparty risk (cf. Duffie and Huang (1996))

We get closed form solution for swap rates as well.

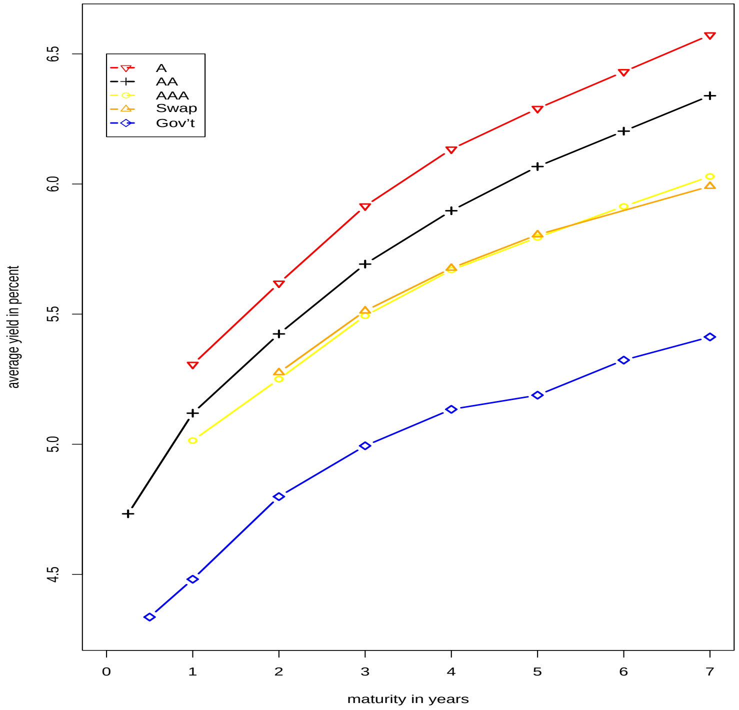
Data

- US government yields, maturities 6 months; 1,2,3,4,5,6,7 years

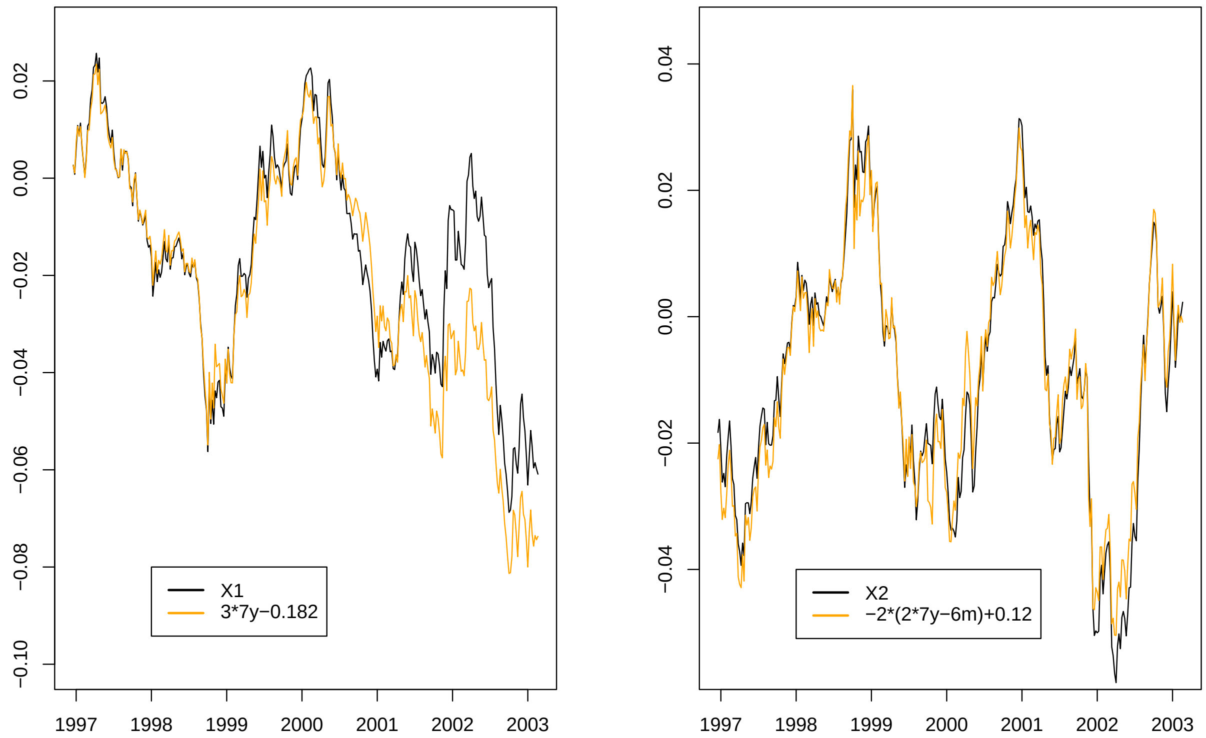
- AAA, AA A, BBB financials (banks also in AAA/AA) 2,3,4,5,6,7 yrs

- US$ swap rates, 2,3,4,5,7 yrs

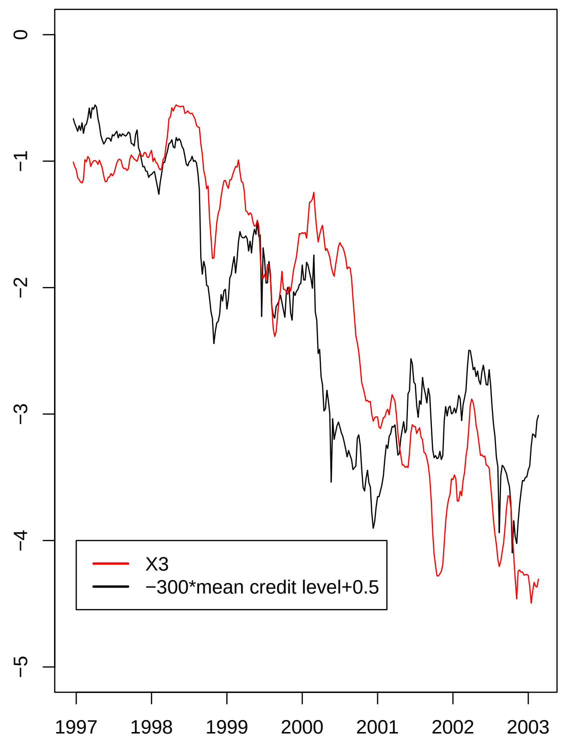
- 3-month LIBOR

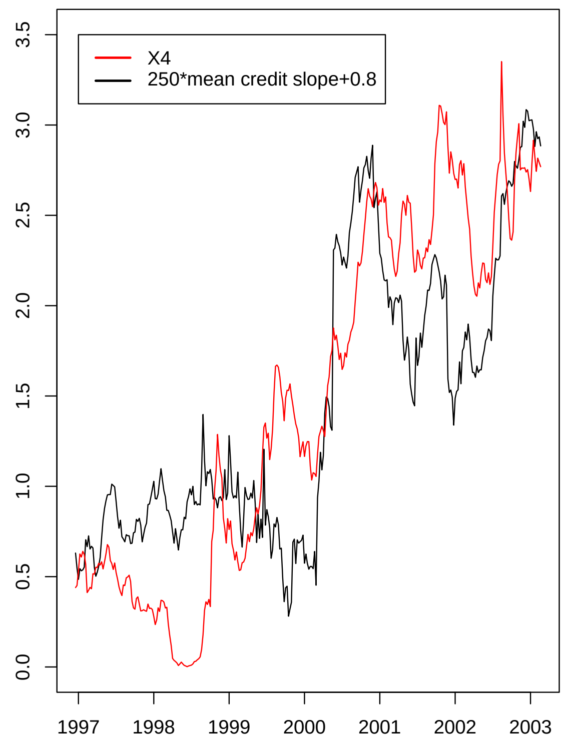


Average curves



The treasury factors





The credit curve factors

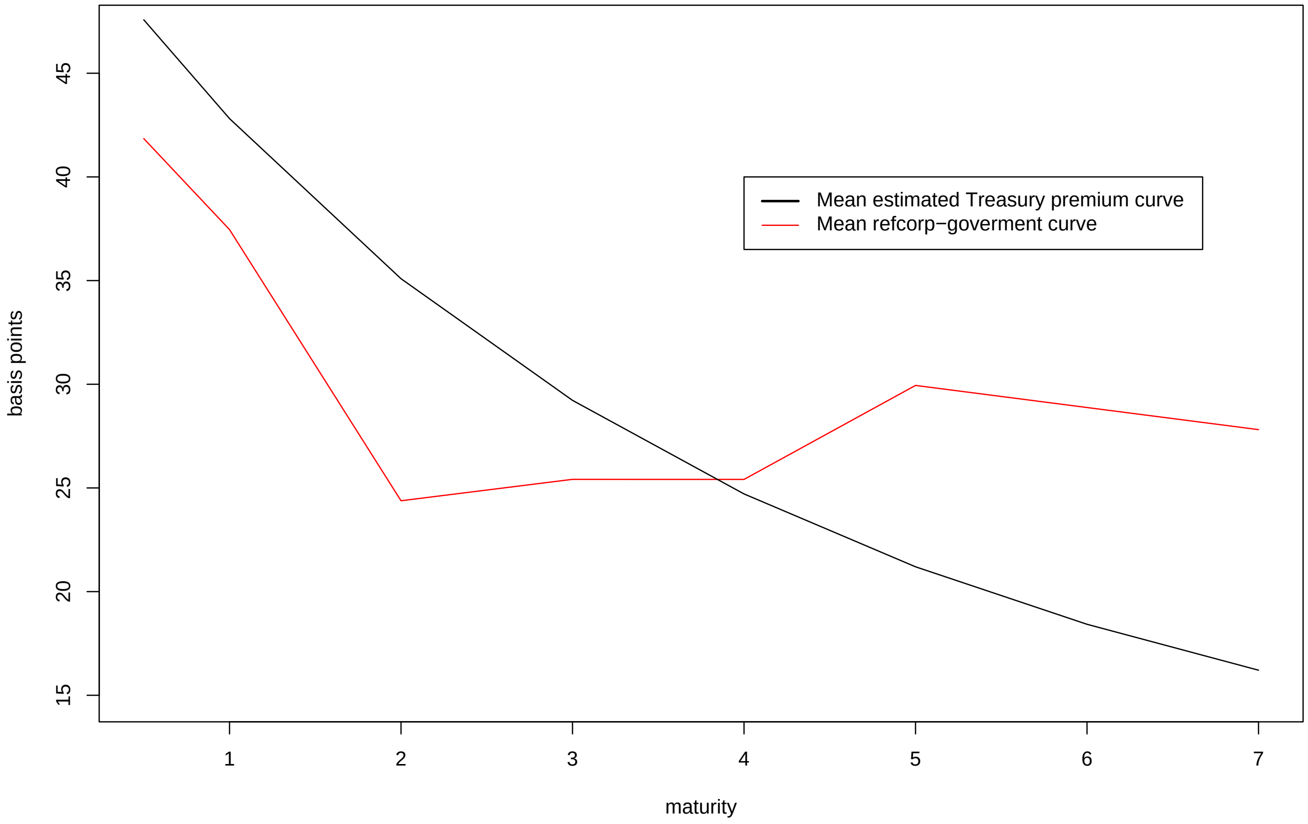
The treasury factor

- We were unable to fit the treasuries and corporate bonds and swaps simultaneously without this factor

- The convenience yield has a term structure

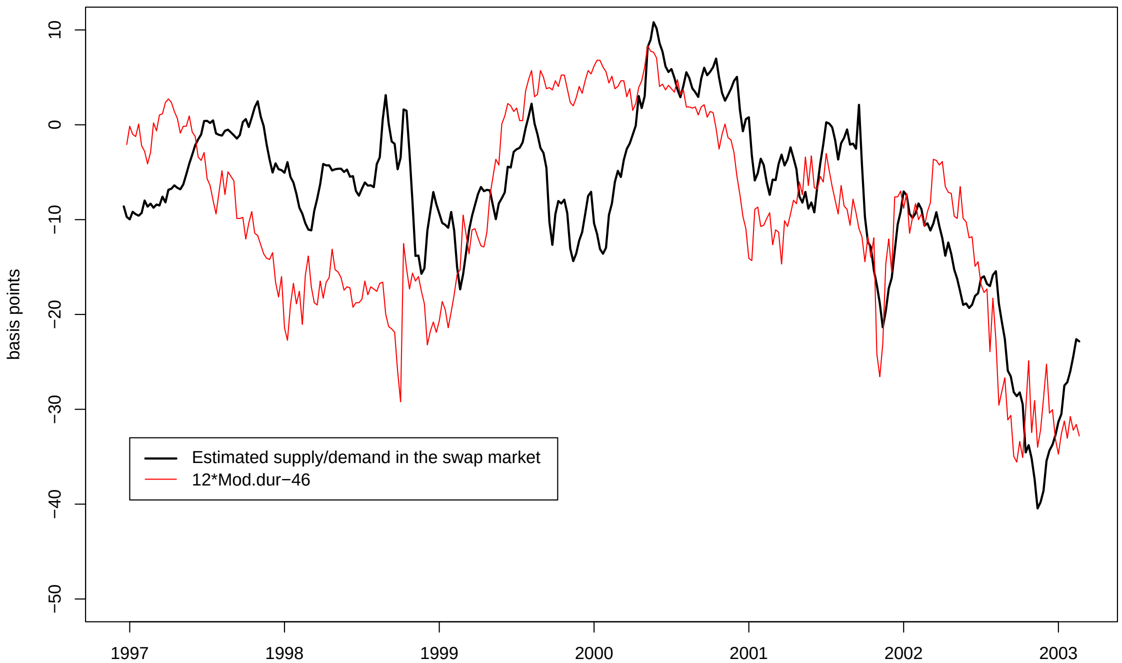
- We compare short end to GC repo - Treasury bill spread and the entire term structure of convenience yields to refcorp - government spreads

- Highest correlation of in 5 yr segment (0.64)

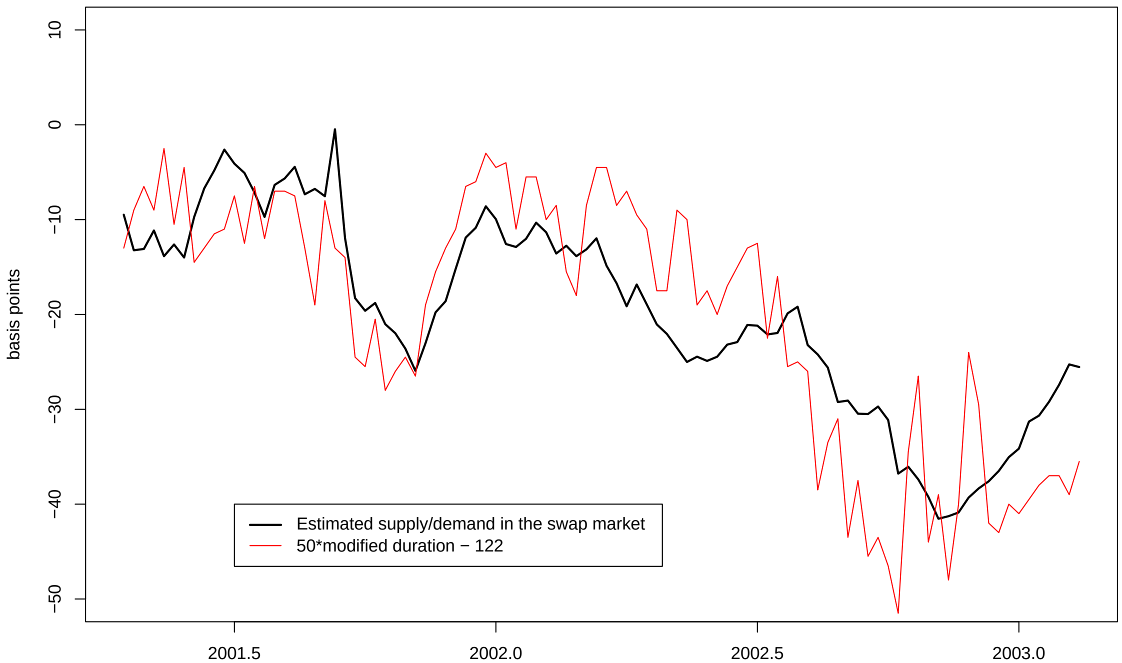

Mean estimated Treasury premium curve
Mean refcorp−goverment curve
basis points
maturity
15
20
25
30
35
40
45
1
2
3
4
5
6
7

MBS duration and the swap factor

- We need separate factor for swap yields
- Separates LIBOR and short corporate curve
- Hedging of agency MBS portfolios seems to be the driving factor
- Example: Interest rates down $\Rightarrow$ duration down
  agencies enter as fixed receivers $\Rightarrow$ swap rate down



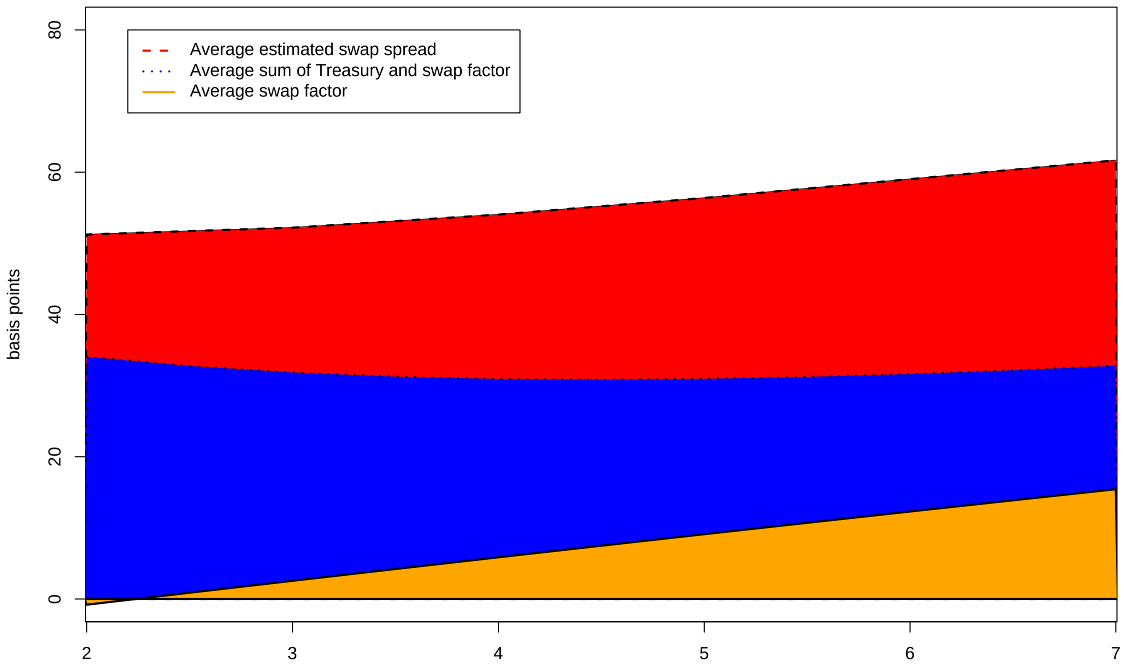
The swap factor and an affine function of Lehman Modified Duration MBS index



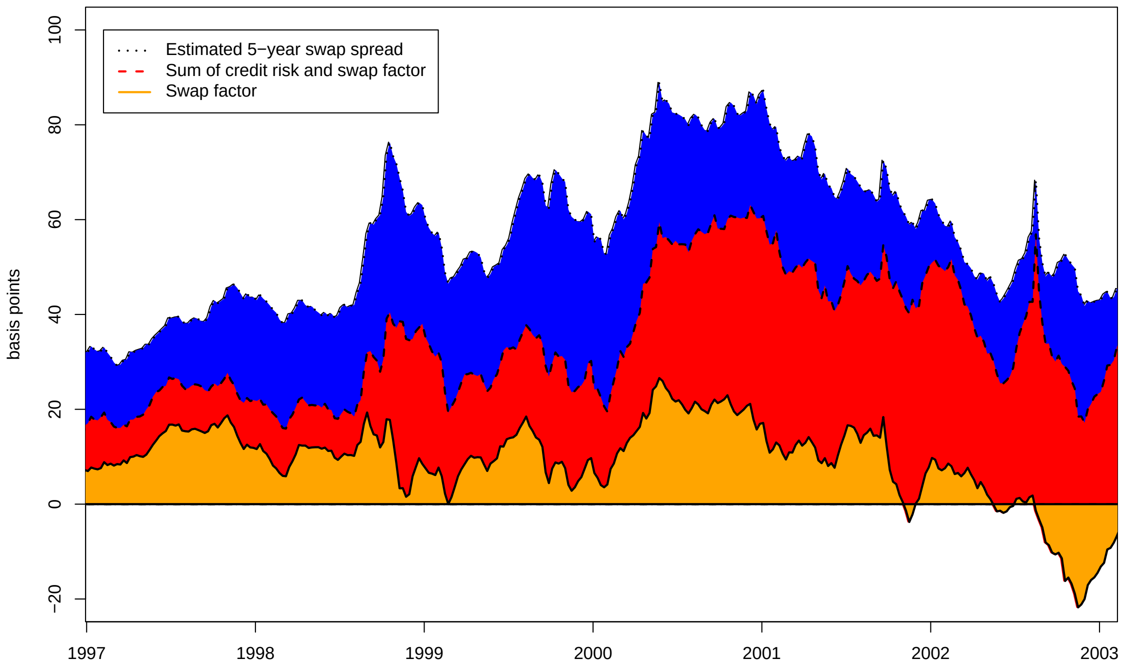
The swap factor and an affine function of Lehman Modified Duration MBS index

Decomposing swap spreads

- We decompose the spread down to treasuries into contributions from swap factor, treasury factor, and from the credit risk component in the underlying LIBOR rate



Decomposing the swap spreads - average curves



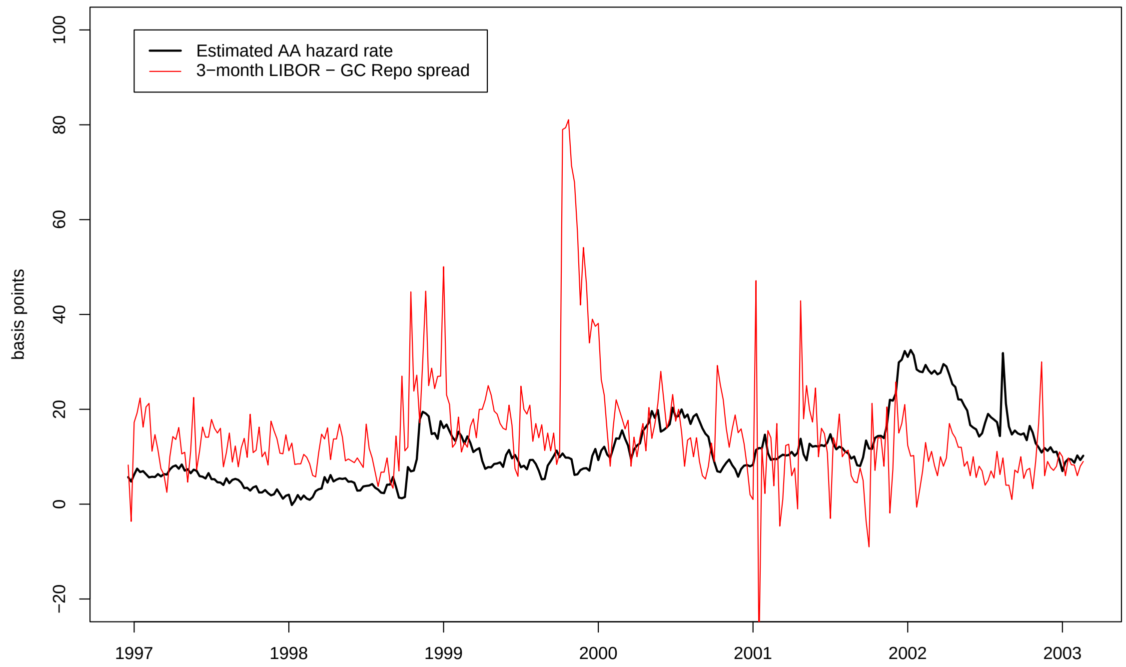
Decomposing the swap spreads - dynamic evolution

The AA short term credit spread

- We compare the short term LIBOR-GC repo spread and find the latter to be too volatile to serve as proxy for short term credit spreads

- Our inclusion of corporate bonds keeps the spread 'in check' making it less volatile and less mean reverting

- This is important for the presence of a credit risk component in long term swap spreads



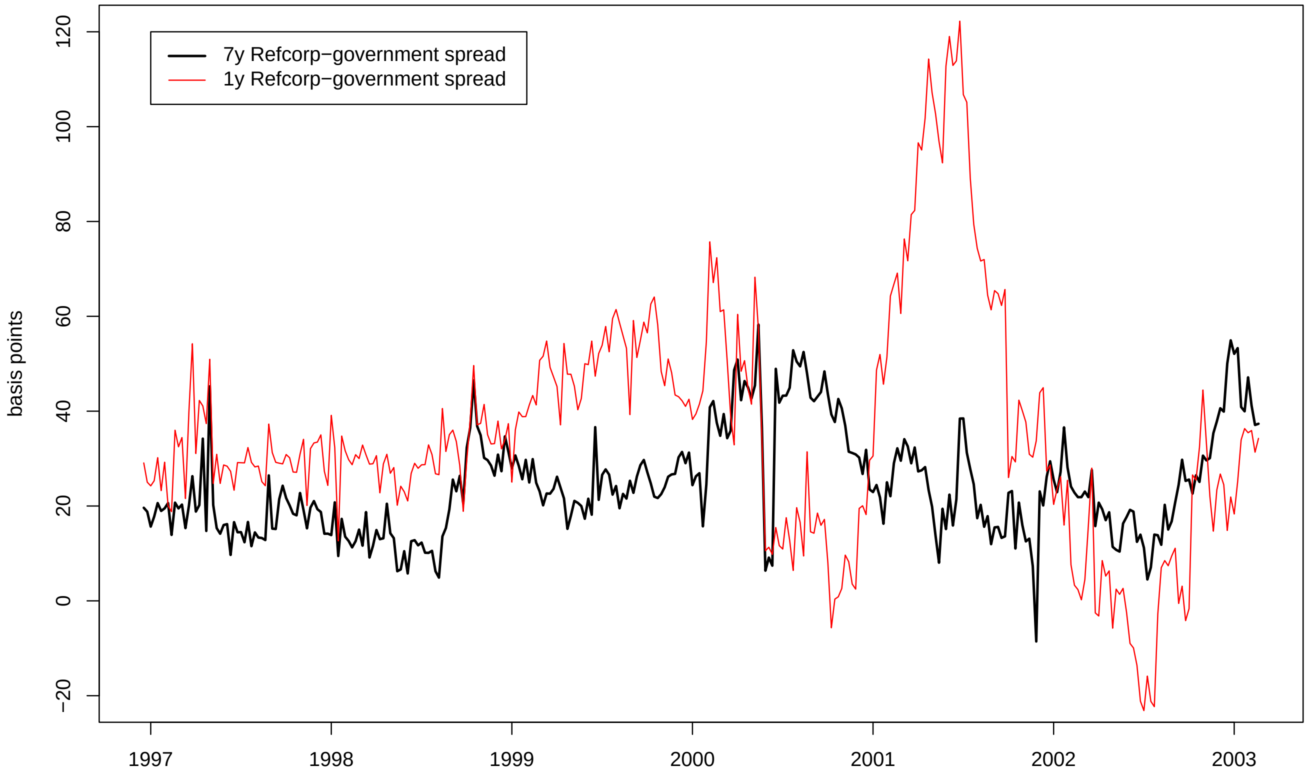
Comparing LIBOR-GC repo rates and our estimated AA short credit spread

Conclusion

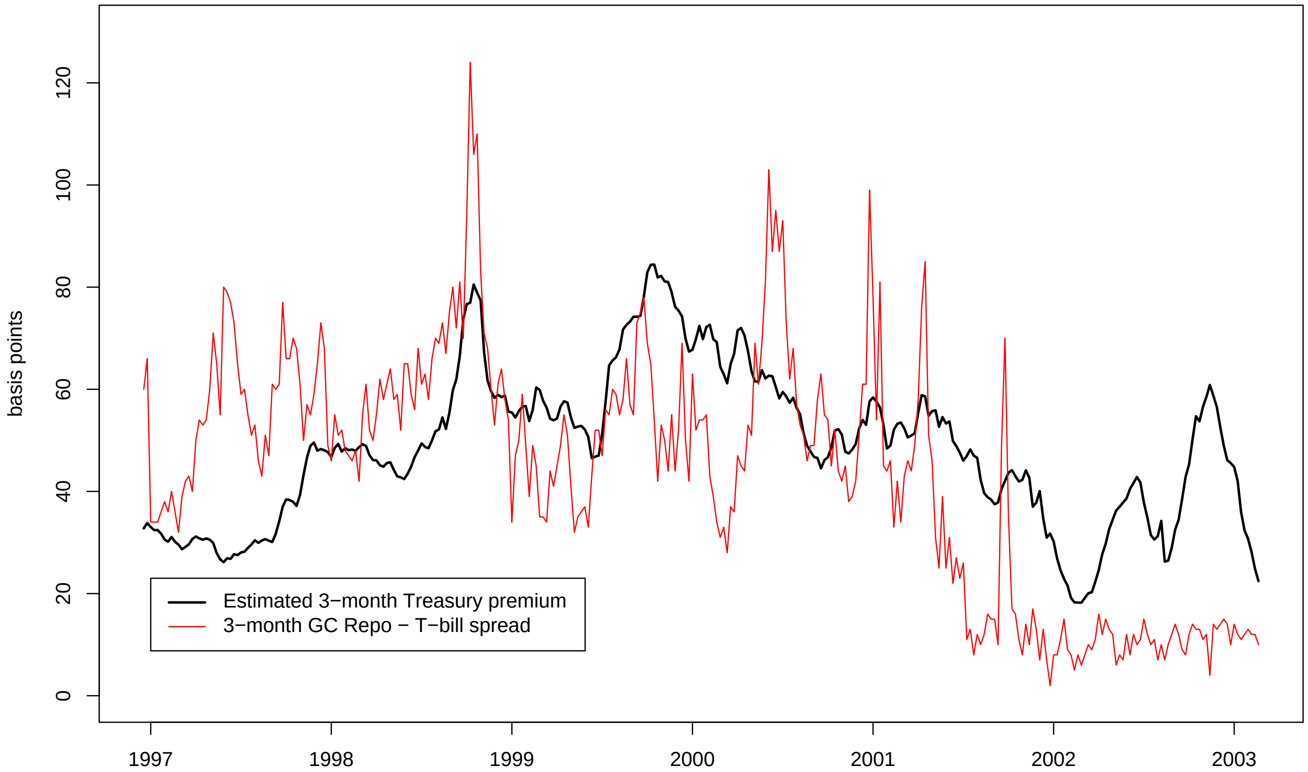
- Obtain a decomposition of swap spreads ¡item identify a strong MBS duration-related component in swap spreads

- LIBOR-GC repo is too volatile as measure of short term AA credit risk?

- Estimating a 'convenience yield' term structure for treasuries and comparing with proxies



7y Refcorp−government spread
1y Refcorp−government spread
basis points
120
100
80
60
40
20
0
−20
1997
1998
1999
2000
2001
2002
2003


basis points
0
20
40
60
80
100
120
1997
1998
1999
2000
2001
2002
2003
Estimated 3−month Treasury premium
3−month GC Repo − T−bill spread