# The Pricing of Unexpected Credit Losses*

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**Abstract**

Why are spreads on corporate bonds so wide relative to expected losses from default? The spread on Baa-rated bonds, for example, has been about four times the expected loss. We suggest that the most commonly cited explanations — taxes, liquidity and systematic diffusive risk — are inadequate. We argue instead that idiosyncratic default risk, or the risk of unexpected losses due to single-name defaults in necessarily "small" credit portfolios, accounts for the major part of spreads. Because return distributions are highly skewed, diversification would require very large portfolios. Evidence from arbitrage CDOs suggests that such diversification is not readily achievable in practice, and idiosyncratic risk is therefore unavoidable. Taking a cue from CDO subordination structures, we propose value-at-risk at the Aaa-rated confidence level as a summary measure of risk in feasible credit portfolios. We find evidence of a positive linear relationship between this risk measure and spreads on corporate bonds across rating classes.

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# 1 Introduction

Spreads on corporate bonds tend to be many times wider than what would be implied by expected default losses alone. These spreads are the difference between yields on corporate debt subject to default risk and government bonds free of such risk. While credit spreads are generally understood to be compensation for credit risk, it has been difficult empirically to establish this link. From 1997 to June 2004, for example, the average spread on Baa-rated US corporate bonds, with a duration of 5 years, was about 182 basis points (at annual rates). Yet, the average expected loss from default on Baa-rated bonds is only 40 basis points (at annual rates). In this case, the spread was over four times the expected loss from default. The wide gap between spreads and expected default losses is what has come to be known as the 'credit spread puzzle' (see, e.g., Collin-Dufresne, Goldstein and Martin (2001), Driessen (2005) and Collin-Dufresne, Goldstein and Helwege (CDGH, 2003)).

Several studies have investigated the determinants of credit spreads or changes in credit spreads. The findings in some recent papers suggest that diffusive risk premia, due to systematic changes in the probability of default, can help account for a portion of spreads (Duffee (1999), Elton, Gruber, Agarwal and Mann (2001), Driessen (2005)). Elton et al. also argue that the differential treatment of taxes on US corporate bonds relative to US Treasury securities can help explain spreads; for example, these authors estimate that 73% of the spread on Aa-rated debt with 5 years to maturity is due to taxes. Another recent strand of the literature has emphasised the impact of relatively poor liquidity in the corporate bond market. For example, Driessen estimates that about 20% of the spread on US corporate bonds is due to a liquidity premium. Further evidence that liquidity premia may be a large component of credit spreads is provided by Longstaff, Mithal and Neis (2004), using data on credit default swaps (CDSs). However, Collin-Dufresne et al. (2001) find that standard indicators of both macroeconomic conditions and liquidity can explain only a fraction of the changes in spreads. Furthermore, the studies noted above obtain their results either from regression analysis or from estimates of reduced-form no-arbitrage models of bond prices. It is well-known that structural models of corporate bond pricing perform as poorly, if not worse, in explaining spreads (see, e.g., Eom et al. (2004) and Huang and Huang (2003)).

In this paper, we offer an explanation that has not been emphasised in the literature. We argue that idiosyncratic default risk is what accounts for much of the difference between spreads and expected losses. This risk has been overlooked because much of the literature has assumed such risk can be diversified away. Jarrow, Lando and Yu (JLY, 2003) show that under certain conditions that permit the construction of diversified portfolios there will, conditionally, be no difference between spreads and expected losses — the default of any particular firm will not command a risk premium. We argue, however, that because default loss distributions are highly skewed, diversification would require portfolios so large that they are, in fact, infeasible to construct. Indeed, we provide evidence that even the most diversified portfolios do not approach the size for which idiosyncratic risk can be ignored. This is consistent with "jump-at-default", or idiosyncratic, risk being priced. The results in Driessen (2005) and Berndt, Douglas, Duffie, Ferguson and Schranz (2004) indicate the presence of jump-at-default risk premia in corporate bond and credit default swap (CDS) spreads, respectively.

If idiosyncratic risk is so important, how is such risk measured by market participants? We propose a measure of risk that is implied by the subordination structure of arbitrage collateralized debt obligations (CDOs). Cash arbitrage CDOs convert the risk of corporate bond portfolios into securities with different levels of risk. This conversion essentially relies on value-at-risk (VaR) calculations. As such, we propose the VaR measure used to create the highly-rated senior tranche in a CDO — which is typically rated Aaa — as the measure for pricing the risk in credit portfolios. This measure not only takes account of the scope for diversification in feasible portfolios, it also satisfies the axioms for a coherent risk measure. We then conjecture that once risk is measured in this way, credit spreads across different corporate bonds will be linearly related to risk. The link between spreads and default probabilities can then be shown to depend in a simple way on a consistent measure of risk and a market price of risk that would apply across different corporate bonds. The use of VaR to measure risk in portfolios is, of course, long established in the risk management profession. However, to our knowledge, no one has as yet proposed VaR as a sufficient statistic for pricing credit risk.

To begin our analysis, in the first part of this paper we reexamine the credit spread puzzle. Using data on option-adjusted spreads from Merrill Lynch over the period 1997-2004, we show that the puzzle may be stronger than previously documented. In addition, unlike in previous studies, we show that the puzzle is present in both US and European corporate bonds. Next, to justify why jump-at-default risk premia may be an important component of spreads, we provide evidence on the size of open-end corporate bond funds and arbitrage CDOs. Our findings suggest that even investors who have a strong incentive to diversify construct only "small" portfolios. We then turn our attention to arbitrage CDOs to understand how they transform risk in corporate bond portfolios and find that they rely on VaR to measure risk. This motivates our proposal for measuring risk in terms of VaR at a Aaa confidence level. Furthermore, we conjecture that this measure of risk leads to a linear pricing equation, and we provide illustrative calculations that show this holds approximately across credit rating classes.

## 2 The Credit Spread Puzzle

One of the puzzles about credit spreads is that they are much larger than expected losses from default; in particular, they are much larger than can presumably be accounted for by the degree, and economic significance, of co-movement between the factors affecting the *probability* of default and the utility of the typical investor. Table 1 presents estimates of the average level of spreads in US and European corporate bonds across rating categories and for three different levels of average duration.[1] These values are computed using option-adjusted spread (OAS) bond indices provided by Merrill Lynch. Data for the United States starts in January 1997, and for Europe in January 1999; both samples end in June 2004.

For US-based corporations, the spreads on Aaa debt have averaged about 50 basis points at shorter durations and 74 basis points at a duration of about seven years. Spreads increase significantly as the rating is lowered down to Baa, and even more so for sub-par investment grade debt, reaching as high as 398 basis points on Ba-rated bonds with a duration of approximately two years. In addition, the term structures are upward-sloping for the higher-rated investment grade bonds, hump-shaped for Baa-debt and downward-sloping for the high yield segment. A qualitatively similar pattern is observed on European corporate bonds, though the average levels of spreads are lower than in the United States.

Table 2 reports estimates of the average intensity (i.e. instantaneous default probability) and expected loss across ratings categories. The intensity is calibrated using the

[1] The notes to the table explain the calculation of average duration.

average five-year-ahead default rate of corporate issuers — which is an estimate of the unconditional five-year default probability — based on data from Moody's. Expected loss is computed as the product of this default probability and a constant rate of loss in the event of default.[2] Loss given default is set equal to one minus the average recovery rate on senior unsecured debt based on Moody's recovery rate data. For the United States, this value is 41.1%; for Europe, it is 18.2%. It is evident that average expected losses are significantly smaller than average spreads, which is robust across ratings and regions.[3]

One useful metric for evaluating spreads is the ratio of average spread to average expected loss — the "Spread Ratio" in Table 2. These ratios are a rough measure of the size of risk-neutral relative to physical default probabilities, and hence they provide evidence on the average size of risk premia associated with idiosyncratic default risk. The very large size of these ratios suggests that such risk premia are a significant component of spreads, in both the United States and Europe. While the spread ratios increase with credit quality, the difference between spread and expected loss — the "Spread Difference" in Table 2 — declines with credit quality. This suggests that, for a given price of risk across rating classes, the measure of risk being priced must increase in absolute value as rating quality deteriorates. Further discussion of these statistics and their implications for modelling credit spreads is taken up below.

Spreads also change over time. Figure 1 plots the spreads on US corporate debt for various rating categories and durations. As shown in the graph, spreads varied considerably over the sample period and they tended to move together across rating categories and average duration, although the shape of the term structure (not shown explicitly) varied through the sample period as well. In particular, in regard to our focus on the puzzle of the large size of spreads, it is important to note that spreads rarely fell below 30 basis points for the most highly rated (Aaa) debt or below 100 basis points for Ba-rated debt.

[2] This calculation implicitly ignores correlation between default probabilities and loss given default.

[3] We also computed expected loss using unconditional ratings transition matrices based on data from Moody's starting in 1985. The results are qualitatively similar.

## 2.1 A Model for Pricing Corporate Bond Portfolios

To help illustrate various issues in the remainder of the paper, we layout here a simple framework for pricing corporate bond portfolios. We consider the class of intensity-based copula models now common in the literature (e.g. Li (2000), Schonbucher and Schubert (2001), Hull and White (2004)). Specifically, for each obligor $i$ of a credit-risky security, default time, $\tau_i$, arrives according to a Poisson process with associated intensity $\lambda_i(t)$. Variation in $\lambda_i(t)$ is assumed to be driven by a vector of Brownian motions $W(t)$, which may contain both common (i.e. aggregate or sector) and firm-specific factors. Correlation in default times is captured using a latent factor/copula approach. For simplicity, we will model correlations in default times using the Gaussian copula, which has become a standard in the market for the pricing of synthetic CDO tranches.[4] Thus, in principle, there are two potential sources of default dependence in our model: first, the *default probabilities* across two obligors can be correlated due to the common dependence of intensities on the risk factors $W(t)$; second, the *default times* across two obligors can also be correlated.

Suppose there are $N$ issuers of default-risky bonds. Let $L_i(t)$ be defined as the percentage loss of face value on bond $i$ in the event of default. Throughout we will assume that the prices of bonds are arbitrage-free, which implies the existence of a stochastic discount factor and an associated equivalent martingale measure $Q$.[5] Let $\xi(t) \equiv E_t\left[\frac{dQ}{dP}\right]$ be the density that defines the change of measure from the physical probability measure $P$ to $Q$. Following CDGH, we assume that the dynamics of $\xi(t)$ are governed by:

$$\frac{d\xi(t)}{\xi(t)} = \sigma_\xi(t) \cdot dW(t) + \sum_{i=1}^{N} J_{\xi,i}(t) dM_i(t) \qquad (1)$$

where $M_i(t)$ is the compensated Poisson process given by $dM_i(t) = d1_{(\tau_i \leq t)} - \lambda_i(t) 1_{(\tau_i > t)} dt$. As is standard in the literature, diffusive risk is represented by $W(t)$, with market prices given by the vector $\sigma_\xi(t)$. The presence of $dM_i$ in (1) means that the pricing kernel is also affected by the default of individual bonds; the market prices on these jump-at-default risks are given by $J_{\xi,i}(t)$.

[4]From a technical perspective, any other form of copula could also be used. Investigating the implications of alternative copulae for the results presented below is part of our ongoing research.

[5]In the current context where markets are incomplete, it is not guaranteed that this measure would be unique.

A non-zero risk premium on default jump risk is equivalent to risk-neutral intensities being higher than their physical counterparts. To be specific, let $\lambda_i^Q(t)$ denote the risk-neutral intensity under the risk-neutral measure $Q$. The relationship between $\lambda_i$ and $\lambda_i^Q$ is (see, e.g., Piazzesi (2003)):

$$\lambda_i^Q(t) = \lambda_i(t)\left[1 + J_{\xi,i}(t)\right] \tag{2}$$

Thus, if $J_{\xi,i} = 0$, then $\lambda_i^Q = \lambda_i$; otherwise, $\lambda_i^Q$ will be larger than $\lambda_i$. In the case that $E_{t^-}\left[J_{\xi,i}(t)\right] = v$, where $v$ is a constant, then

$$\frac{\lambda_i^Q(t)}{\lambda_i(t)} = 1 + v$$

or, taking unconditional expectations,

$$\frac{E\left[\lambda_i^Q\right]}{E\left[\lambda_i\right]} = 1 + v \tag{3}$$

## 2.2 Explanations of the Credit Spread Puzzle

Several explanations of the credit spread puzzle have been offered in the literature. Here we briefly review the arguments and evidence regarding the role of taxes, liquidity premia, diffusive risk premia and jump-at-default risk premia and their effects on spreads.

### 2.2.1 Taxes

In the United States, corporate bonds are subject to taxes at the state level, whereas Treasury securities are not. Since investors compare returns across instruments on an after-tax basis, arbitrage arguments imply that the yield on corporate debt will be higher to compensate for the payment of taxes. Maximum marginal tax rates on corporate bonds vary roughly from 5 to 10% across states. Taking account of the deduction of state taxes from federal tax, Elton et al (2001) use a benchmark tax rate of 4.875% to find that taxes can account for 28–73% of spreads, depending upon rating and maturity. Using a different sample and methods, Driessen (2005) finds that taxes may account for 34–57% of spreads. Note that, since taxes are applied to the level of coupons or yields rather than spreads, they would tend to account for a larger proportion of spreads in the higher-rated issuers.

One argument against attributing a significant role to taxes is that the relevant tax rate depends upon where the marginal investor in corporate bonds resides. Since tax rates in some jurisdications are trivial, it is arguable that taxes have no impact at all on spreads. If this were not the case, all else equal, it would be very profitable on a *risk-adjusted basis* for an investor to take long positions in corporate bonds simply by residing in a low tax jurisdiction. A second argument against taxes as an explanation of the credit spread puzzle is that there is less systematic difference regarding the tax treatment of corporate and government bonds in European countries, yet a spread puzzle appears to be present on European corporate bonds as well.

### 2.2.2 Liquidity Premia

Even in the United States, most corporate bonds trade in relatively thin markets. The market for corporate debt is less mature and even less liquid in Europe. This means that it is typically more costly to undertake transactions in these instruments than in equities and government bonds, and investors must be compensated for this. For example, Schultz (2001) estimates that round-trip trading costs in the US corporate bond market are about 27 basis points. More generally, there can be uncertainty about the liquidity (or illiquidity) of a given bond at a given time, and investors might also require a premium to bear this risk.[6] Indeed, several recent studies have argued that liquidity premia may be the next most important component of spreads after taxes. Driessen (2005) estimates that liquidity premia account for about 20% of spreads, with Perraudin and Taylor (2003) obtaining even larger estimates.[7]

Measuring liquidity premia is tricky. Various proxy measures for the liquidity of an instrument exist, such as turnover, number of transactions and bid-ask spreads. However, one problem is that different theories of market liquidity sometimes give opposite predictions for the behaviour of these variables. Moreover, at a deeper level, it is likely that default and liquidity risk are linked together, which means it can be difficult in practice to identify separate liquidity and risk premia terms.

In terms of the corporate bond market specifically, many issues trade little shortly

[6] This is an example of liquidation risk; see Duffie and Ziegler (2003).

[7] See also Delianedis and Geske (2001), Janosi, Jarrow and Yildrim (2001) and Dignan (2003), amongst many others.

after issuance because of holding restrictions on key institutional investors or the incentives on fund managers to construct benchmark portfolios. Yet, buy-and-hold investors are unlikely to require a large premium for liquidity. For instance, institutional investors tend to buy and hold the stock of available Aaa-rated bonds because there are so few firms with this rating to begin with. As in the case of taxes, the impact of market liquidity on the price of a given bond will depend upon the marginal investor. If the marginal investor is a buy-and-hold investor, it is hard to see how illiquidity could command much compensation.

Some evidence that the recent literature has overstated the role of liquidity premia in explaining the size of spreads can be gleaned from a comparison of spreads on corporate bonds and CDSs. The difference between the CDS spread and the spread on a par floater bond of the same issuer is known as the default swap basis. In an idealised case, the basis is equal to zero in the absence of arbitrage opportunities. Figure 2 plots the basis for the majority of entities included in the Dow Jones Trac-x European High Grade Series 2 index. As can be seen in the graph, the European basis has mostly been positive since the CDS market became much more liquid in 2002, averaging 7.2 basis points over the period shown. In practice, there are several reasons why the basis may be either positive or negative and can change over time. One reason the basis might be non-zero is due to greater liquidity in one of the instruments. However, since the Trac-x index was based on the most actively traded entities in the CDS market, it is unlikely that a large positive basis could arise due to a relative liquidity premium in CDS spreads. Thus, the fact that the basis on European Trac-x names has been positive suggests that the liquidity premium in European corporate bonds has been negligible.[8]

### 2.2.3 Diffusive Risk Premia

How important is the diffusive risk component in expected excess returns (or, equivalently, spreads)? An answer to this requires having estimates of both the volatility of returns ($\sigma_m$) and the market price of diffusive risk ($\sigma_\xi$). Table 3 reports measures of the volatility of returns on the Merrill Lynch indices. The estimates of volatility are unconditional estimates of the monthly volatility of total returns (expressed as annualised percent). Values of $\sigma_\xi$ are more difficult to obtain. An indirect approach to evaluating

[8] See Longstaff et al. (2003) for contrasting evidence based on CDS spreads for US firms.

the importance of diffusive risk premia is to compute Sharpe ratios and judge whether they are "reasonable". Estimates of expected excess returns and the corresponding Sharpe ratios are shown in Table 3 as well. Expected excess returns are calculated as spread minus expected loss plus the (ex post) average slope of the term structure.[9] As shown in the table, the Sharpe ratios, across ratings and maturities, are roughly similar in size to those found in the equity literature. However, it is important to point out that the measures of volatility in Table 3 are probably upward biased. Since the estimates are based on monthly changes in bond prices, they also incorporate factors other than diffusive volatility that change over time and affect bond prices (e.g. the components of idiosyncratic risk premia). Thus, the true Sharpe ratios on corporate bonds are likely larger than what has been observed for equities.

Previous studies have also attempted to estimate what percentage of spreads can be attributed to diffusive risk. Elton et al. (2001) run regressions of changes in spreads (after subtracting expected loss and tax components) on the Fama and French (1993) risk factors. Using their results, it is possible to estimate what portion of spreads are explained by these proxies for diffusive risk. Depending upon maturity and rating, the Fama-French factors can account for 19-41% of spreads. By contrast, Driessen (2005) decomposes spreads into several components by estimating an intensity-based no-arbitrage model using a dataset of prices on individual US corporate bonds. He finds that diffusive risk accounts for a smaller portion of spreads (a maximum of 19%).

#### 2.2.4 Jump-at-Default Risk Premia

In JYL, the assumptions required for conditional diversification to hold imply that $J_{\xi,i}(t) = 0$ in (1). Otherwise, default events will be priced. By making some simplifying assumptions in the pricing of corporate bonds, it is possible to obtain an estimate of the average value of $J_{\xi,i}(t)$ using data on average spreads, default rates and recovery rates.

Suppose $w_i(t)$ is the recovery rate on bond $i$ in the event of default at time $t$.

[9]For the United States, the term structure slope is computed by taking the difference between the yield on the Treasury note with the relevant maturity and the 3-month Tresasury bill rate; for Europe, yields on German bunds are used. For both economies, the yields data is for zero-coupon constant maturity securities.

Under Duffie and Singleton's (1999) "Recovery of Market Value" (RMV) assumption on recovery in the event of default, $w_i(t) = V_i(t^-, T)\left[1 - L_i^Q(t)\right]$, where $V_i(t^-, T)$ is the price of a zero-coupon credit-risky bond just prior to default and $L_i^Q(t)$ is the (risk-neutral) loss rate. In this case, the price of a zero-coupon credit-risky bond is given by

$$V_i(t,T) = E_t^Q\left[\exp\left(-\int_t^{t+T}\left(r(s) + \lambda_i^Q(s)L_i^Q(s)\right)ds\right)\right] \tag{4}$$

where $r(t)$ is the instantaneous riskfree rate. If $\lambda_i^Q(s)$ and $L_i^Q(s)$ are the constants $\lambda_i^Q$ and $L_i^Q$, then (4) becomes

$$V_i(t,T) = \exp\left(-(T-t)\,\lambda_i^Q L_i^Q\right) E_t^Q\left[\exp\left(-\int_t^{t+T} r(s)ds\right)\right] \tag{5}$$

The second term on the right-hand side of (5) is the price of a zero-coupon default-free bond with the same time to maturity as the corporate bond. Thus, if $S_i(t) \equiv y_i(t) - y_G(t)$ is the spread on bond $i$, where $y_i(t)$ and $y_G(t)$ are the yields to maturity on bond $i$ and the default-free bond, respectively, (5) implies

$$S_i(t) = \lambda_i^Q L_i^Q$$

Thus,

$$\frac{\lambda_i^Q}{\lambda_i} = \frac{S_i(t)}{\lambda_i L_i^Q} \tag{6}$$

The denominator on the right-hand side of (6) is approximately equal to expected loss. Thus, by (3), the Spread Ratio roughly equals $1 + \nu$. The statistics in Table 2 suggest that $v$ varies with rating (and maturity), and the differences across ratings are large, ranging from as high as 625 for Aaa-rated bonds! down to 2 for Ba-rated bonds at the five-year maturity.

Alternative estimates of $v$ have also appeared in the literature recently. Driessen (2005) obtains estimates of $v$ ranging from one to five. This is based on the ratio of his estimates of average risk-neutral intensities to the average historical one-year default probability on the universe of investment grade firms rated by Moody's. Furthermore, Driessen finds that anywhere from 10 to 37% of spreads can be accounted for by jump-at-default risk. In other work, Berndt et al. (2004) estimate $v$ to be slightly larger than one using data on CDS spreads and EDFs from Moody's KMV to proxy for physical intensities. CDGH argue that jump-at-default risk premia can only account for a negligible portion of the spread on US investment-grade bonds. They reach their conclusions by assuming that: (a) diffusive risk is (relatively) high; and (b) investors can form large portfolios (e.g. about 1000 corporate bonds). Specifically, their estimates of excess returns and diffusive volatility are 2.7% and 8%, respectively, and they assume that the (modified) instantaneous Sharpe ratio is 0.3. This implies that non-diffusive risk premia — in their context, the sum of jump-at-default and contagion risk premia — accounts for only 27% of excess returns. Furthermore, their calibration implies that the jump-at-default risk premium is only 0.003%.

Ultimately, jump-at-default risk premia will only be a significant portion of spreads if credit portfolios only have exposures to a small number of names in practice. A key question, then, is: how large are corporate bond portfolios that investors can actually hold? One way to determine the feasible size of corporate bond portfolios is to look at the size and composition of the Merrill Lynch indices themselves. In the US indices, there were a total of 752 investment grade issuers on 4 August 2004, although the vast majority of these are rated A and Baa.[10] Notably, there are only 30 and 50 firms with a Aaa and Aa rating, respectively. The fact that these numbers are decreasing with credit quality is consistent with larger spread ratios for higher ratings categories and an idiosyncratic risk premium being an increasingly important component of spreads for higher rated firms.

In reality, there are few investors, if any, who hold the universe of firms in the Merrill Lynch indices (or, for that matter, indices from other dealers, e.g. Lehman Brothers). Instead, it is perhaps more appropriate to examine the structure of open-end corporate bond funds. Table 4 reports summary measures on the five largest investment grade and five largest high yield funds domiciled in the United States (chosen by assets), which have at least 70% of their portfolio invested in corporate bonds. The values in the table were determined on 24 August 2004. Focusing on investment grade funds, the largest is Vanguard Intermediate-Term Investment, which is composed of 385 distinct issuers; the other top four funds have 130-204 names in them. Notice, however, that the number of corporate bond issuers could be much smaller. As shown in the table, only 81.35% of the Vanguard fund, for example, is corporate bonds. If we were to pool the top five funds, the total number of distinct issuers would be 685, but, again, the

[10]The Merrill Lynch indices are rebalanced on a monthly basis, so the number of issues and issuers included in the indices may change over time.

holdings corresponding to each of these are not necessarily corporate bonds.

While the sizes of bond indices and bond funds seem to suggest that credit portfolios can gain exposures to as many as 700 distinct issuers, many of the securities underlying these products are not very liquid. In reality, an investor trying to construct their own corporate portfolio may find it very difficult to achieve a similar level of diversification with those securities that are actively traded. Arguably a better place to look for the portfolios of marginal investors is the market for collateralised debt obligations (CDOs), particularly arbitrage CDOs, since this has been a faster growing segment of the credit market. Arbitrage CDOs are vehicles for securitisation that rely on lower-rated debt securities as collateral and issue several tranches of notes as liabilities, the bulk of which are typically Aaa-rated securities. Arbitrage CDOs offer interesting insights for our purposes because they are structured precisely to exploit credit spreads that are wide relative to expected losses, and their success depends on how well they can diversify default risk. The extent to which they do diversify would then be evidence of what kinds of portfolios are attainable in practice.

Table 5 contains statistics on the collateral pools of cash arbitrage CDOs, based on data from Moody's CDO indices. Most important for our purposes is the diversity score, which is intended to measure the size of the collateral pool in terms of the equivalent number of obligors with independent default times (i.e. it strips out the effects of correlations). Across the number of deals reported by Moody's, the mean diversity score is 50.69 and the maximum is 64. Moreover, the long ramp-up periods typically required for assembling the collateral pool suggest that the average-sized portfolio actually being held at any given time by investors could be significantly smaller than the final size of the portfolio.

# 3 Lessons from Arbitrage CDOs

Arbitrage CDOs not only provide evidence on the size and structure of corporate bond portfolios. The risk structure of the liabilities of CDOs sheds light on the pricing of credit risk itself. This section discusses how the risk of CDO collateral pools is determined. The next section discusses implications for pricing.

As mentioned above, arbitrage CDOs are particularly interesting for our purposes because they are structured precisely to exploit credit spreads that are wide relative to expected losses. Indeed, CDO managers have been described as "seekers of spread", and issuance of arbitrage CDOs tends to rise when credit spreads are wide.[11] One way to interpret the term "arbitrage" in "arbitrage CDOs" is to think of CDO managers as essentially pursuing a strategy of arbitrage between physical and risk-neutral intensities. We will show that the success of this strategy depends on how well the CDO structure diversifies default risk. In the previous section, we reported evidence that the collateral pools of arbitrage CDOs are small in size, around 50-100 names. In this section, we will infer from the tranche structure of arbitrage CDOs what extent diversification is attainable in practice, and, as a corollary, what amount of idiosyncratic risk is faced by holders of corporate bond portfolios.

At the same time, the fact that CDOs are able to transform debt collateral of various credit ratings into a set of tranched securities, with the most senior tranche almost always being a highly-rated note with a Aaa rating, suggests that the credit market has developed a common yardstick for measuring credit risk that applies across the different ratings of debt.

## 3.1 An Arbitrage Strategy?

To understand the strategy of arbitrage CDOs, consider how a CDO manager might employ a collateral pool of Baa-rated bonds. The estimates in Table 2 suggest that such bonds would each have a physical default intensity of 0.7% a year and a recovery rate of 41%. In this case, the expected loss will amount to 40 basis points in annual terms. Suppose also that the credit spread paid on these bonds is 180 basis points. If the collateral pool is perfectly diversified, the CDO manager will not need to be concerned about unexpected losses from default (abstracting from diffusive risk). By setting aside 0.4% of the collateral pool to cover expected losses, the amount of the remaining collateral will constitute a portfolio that has minimal default risk. The manager can then issue Aaa-rated bonds against this essentially risk-free portfolio. In this example, it is the spread differential between Baa-rated and Aaa-rated bonds minus the proportion of over-collateralisation. Here, the 0.4% over-collateralization will just equal the losses from default. If the spread on Aaa-rated bonds is 70 basis points, the

[11] See BIS (2004), pp. 119-120.

arbitrage gain will be 70 basis points (110 basis points for the spread differential and 40 basis points for over-collateralisation). For what is an essentially riskless arbitrage strategy, this is certainly an extraordinarily large gain.

In practice, of course, we cannot expect such arbitrage opportunities to be readily available. What prevents CDO managers from consistently making such large gains? The short answer is that there are no perfectly diversified collateral pools and the manager must therefore face idiosyncratic risk in the form of unexpected losses from default. The arbitrage strategy is not truly riskless.[12] To illustrate this point, Table 6 shows the liabilities structure of a typical CDO — the Diamond Investment Grade CDO, Ltd. I — as well as market-wide averages, based on data from JP Morgan Chase. The collateral in this particular CDO is a mix of different types, but is mainly composed of Baa bonds. The total number of names in the collateral pool is 136. However, the diversity score assigned by Moody's suggests that the possibility of default correlations would make the effective number of independent obligors closer to 60 (the role of correlations will be discussed further below). The loss distribution for a portfolio of 60 independent obligors assigns a significant probability to large unexpected losses, and the portfolio is therefore not well diversified. This particular CDO issued notes in four tranches, with the senior Aaa tranche amounting to 83% of the total face value. The "equity" portion of 4% plus the mezzanine tranches of 13% represent the overcollateralization required to protect the Aaa tranche from losses from defaults in the collateral pool. Since the expected loss is small, most of the required overcollateralization represents coverage for unexpected losses. Hence, the proportion of overcollateralization is a measure of the idiosyncratic risk stemming from the limited degree of diversification.

[12] Conversations with practitioners suggest that one of the main appeals of CDOs is that investors value the portfolio expertise implicitly offered by the CDO manager. This is yet another reason to expect we should observe large CDO collateral pools in reality. CDO managers arguably earn profits by also exploiting liquidity premia in corporate bonds, i.e. by being more efficient at assembling collateral than the typical investor.

# 4 Pricing Risk in Credit Portfolios

To explain credit spreads, we need to specify a mapping between physical and risk-neutral default intensities. This mapping will entail a risk measure, such that greater risk leads to a wider spread. In this section, we will propose a risk measure for credit portfolios and investigate whether this measure can lead to a positive linear relationship between risk and return.

## 4.1 Measuring Risk

In portfolio analysis, the most common measure of risk is the *volatility* of returns, or, equivalently, the variance of returns. The advantage of this measure is that it is easy to compute. However, volatility suffers from the problem that it gives the same weight to the "upside" as to the "downside." Hence, in general, it is contrary to our intuition about risk, which is about "bad" things happening. Nonetheless, if investment returns were symmetrically distributed, volatility would be a good summary statistic for the downside. In the case of credit portfolios, however, investment returns are not symmetrically distributed. For such portfolios, the possibility of large losses from default gives us return distributions that are negatively skewed, and here volatility would be inappropriate as a measure of risk.

How would we capture the downside risk of a credit portfolio? One alternative is to turn to the family of fairly complicated statistics known as lower partial moments. An example of these is *expected shortfall*, which measures the expected loss on a portfolio conditional on losses having passed some threshold in the tail of the loss distribution.[13] Among risk managers, the most widely used measure by far is a much simpler one called *value-at-risk* (VaR), which is the amount of loss that is exceeded at a given confidence level. Indeed we find that this measure is increasingly becoming the standard for measuring risk in credit portfolios.[14] It is from this particular risk measure that we will draw implications for pricing.

[13] Expected shortfall has been investigated by Artzner et al. (1999), O'Kane and Schloegl (2002) and Albanese and Lawi (2003), among others.

[14] Other studies to examine VaR as a portfolio risk measure include Alexander and Baptiste (2003) in the case of equities and O'Kane and Schloegl (2002) and Albanese and Lawi (2003) in the case of credit.

Before we turn to pricing, however, we would like to comment on two objections that have been leveled against VaR as a measure of risk. The first is that the choice of confidence level is arbitrary. Should it be 99% or 99.95%? Different risk managers would choose different confidence levels. The second objection is that, in general, VaR is not a coherent risk measure. As defined by Artzner et al (1999), a measure $m(X)$ is coherent if it satisfies four axioms: (a) homogeneity: for any number $c > 0$, $m(cX) = cm(X)$; (b) monotonicity: $m(X) \geq m(Y)$ if $X \geq Y$; (c) risk-free condition: $m(X + k) = m(X) - k$, for constant $k$; and, (d) subadditivity: for any two payoffs, $X$ and $Y$, $m(X+Y) \leq m(X)+m(Y)$. In general, VaR satisfies the first three axioms but not subadditivity.[15] This last axiom is important: unless a risk measure is subadditive, it will not recognize the advantage of diversification.[16]

Participants in credit markets seem to have found a way around these objections. With regard to the objection of arbitrariness, the rise of CDOs has led to a convergence of VaR confidence levels to the one that is consistent with the survival probability of Aaa credits. This confidence level is implicit in the subordination structures of CDOs. While there are variations in the way credit rating agencies assess CDO structures, the basic idea is the same. To properly protect the Aaa tranches, which form the largest part of the structures, the combined size of the subordination tranches is set equal to the VaR of the collateral pool at the confidence level of the Aaa survival probability.[17] Hence, the choice of confidence level is no longer arbitrary. Conveniently enough, in settling on a common VaR confidence level, CDO managers have also found a solution to the problem of coherence. The Aaa survival probability implies a rather high confidence

[15] Duffie and Singleton (2003) provide the following example of how a VaR risk measure violates subadditivity. Suppose $X$ and $Y$ are independently and identically distributed payoffs on two loans, each of which pays 100 with probability 0.994 and otherwise pays zero. The VaR risk measure at the 99% confidence level then gives $m(X) = m(Y) = 0$. Yet, when we consider a portfolio that combines half of each loan, we obtain $m(X/2+Y/2) = 50$. The risk measure deems the more diversified portfolio to be riskier.

[16] In addition, as noted by Yamai and Yoshiba (2005), VaR does not take account of losses greater than the VaR level, which may occur during periods of market stress.

[17] Strictly speaking, this VaR approach in deciding subordination corresponds to the PD approach in the way Standard & Poor's and Fitch assign ratings; for descriptions of these methodologies, see Standard & Poor's (2002) and Bund et al. (2003), respectively. An expected shortfall approach corresponds more closely to the way Moody's assigns ratings (see Yoshizawa and Witt (2003)).

level — in our estimates, this is effectively a confidence level of 99.999% at the one-year horizon. At a confidence level this high, the VaR risk measure will be subadditive for any two portfolios consisting of lower rated names.

Hence, we propose to measure credit risk based on the VaR of a credit portfolio at the confidence level of the Aaa survival probability. So that the risk measure properly takes account of the size of the portfolio, we specify it as the ratio:

$$\omega_{Aaa}(N, \lambda, \rho) \equiv \frac{VaR_{Aaa}(N, \lambda, \rho)}{N} \tag{7}$$

For purposes of illustration, we construct portfolios of equal-sized bonds for each of $N$ names with the same default intensity $\lambda$ and a common pairwise default time correlation $\rho$.

In the case of CDOs, the amount of subordination is effectively determined by rating agencies: they calculate the amount that will be sufficient to protect the higher rated tranches against defaults in the collateral pool at probabilities consistent with the ratings of those tranches. If, for simplicity, we assume zero recovery from default, then the required overcollateralization for a collateral pool consisting of $N$ equally weighted names is given by:

$$k^* \equiv \min k \quad s.t. \quad 1 - F_B(N, k, \lambda_i, \rho) \leq \lambda_{Aaa} \tag{8}$$

where $F_B(N, k, \lambda_i, \rho)$ is the cumulative distribution for $k$ defaults out of the $N$ bonds, $\lambda_i$ is the default intensity for each individual name in the collateral pool and $\lambda_{Aaa}$ is the default intensity for the highly-rated senior tranche (for all practical purposes, the Aaa tranche).

As an illustration, assume that default intensities are independent across the $N$ bonds in the collateral pool. Then $F_B$ will be a cumulative binomial distribution.[18] Figure 3 shows the required overcollateralization in proportion to the size of the portfolio, for each of three different values of $\lambda_i$: one corresponding to a Ba-rated pool, one to Baa-rated pool and one to an A-rated pool. The higher the default intensity of the collateral pool, the higher $k^*/N$ will be. Moreover, this ratio is a declining function of the number of names in the collateral pool. Although not shown in the figure, it can

[18] This procedure for calculating the loss distribution is called the Binomial Expansion Method, which has been used by Moody's as part of its methodology for rating several types of CDO structures. See Cifuentes and O'Connor (1996) for further details.

be surmised that as $N$ gets very large, the ratio approaches from above the difference in probabilities, $\lambda_i - \lambda_{Aaa}$.[19]

The relationship between risk and diversification is clearly evident in Figure 3. The figure shows that the bigger the collateral pool, the smaller the overcollateralization ratio and the smaller the risk faced by the CDO manager. The fact that the overcollateralization ratio continues to decline with the size of the collateral pool means that arbitrage gains also increase. The CDO manager clearly has a strong incentive to increase the size of the collateral pool or, more precisely, the number of independent names in the pool. In spite of this incentive, however, the typical arbitrage CDO structured on investment grade assets contains only about 100 names in its collateral pool, resulting in an average diversity score of only about 51 names (Table 5). In the case of high-yield collateral, the average diversity score is only 42 names. Only a few CDOs have had more than 200 names. Conversations with market participants suggest that it can take many months for a CDO manager to assemble the collateral for a given structure. It appears that beyond a few benchmark bonds, the cost of searching for additional names rises sharply, and at some point it simply becomes infeasible to construct a more diversified portfolio. Hence, full diversification is not achieved even by those investors who would have the most to gain.[20]

An important point to note is that $k^*$ is just the VaR of the collateral portfolio with the confidence level set at $F_B(N, k, \lambda_i, \rho) = 1 - \lambda_{Aaa}$: $k^* = VaR_{Aaa}(N, \lambda_i, \rho)$. In other words, the confidence level is specified to be such that the tail probability $1 - F_B(N, k, \lambda_i, \rho)$ is the Aaa default intensity. The ratio $\omega_{Aaa}$ in (7) can then be interpreted as a measure of the risk of a portfolio with $N$ equally weighted names, each of which has a default intensity $\lambda_i$, and where the dependence of $\omega_{Aaa}$ on $\lambda_{Aaa}$ is

[19] Strictly speaking, the overcollateralisation ratio does not uniformly decline with respect to $N$. This is because $k^*$ increases with $N$ in discrete steps. That is, when $k^*$ increases by one, say, as the portfolio size increases from $N$ to $N+1$, it will be the case that, since $k^* < N$, $\omega_{Aaa}(N+1, \lambda_i, \rho) = \frac{k^*+1}{N+1} > \frac{k^*}{N} = \omega_{Aaa}(N, \lambda_i, \rho)$. More generally, for loss given default $L < 1$, $\omega_{Aaa}(N+1, \lambda_i, \rho)$ may or may not be larger than $\omega_{Aaa}(N, \lambda_i, \rho)$ when the critical value $k^*$ changes. In practice, an adjustment for this type of granularity would be needed to make VaR, as a proportion of portfolio size, a monotone decreasing function of $N$.

[20] Other factors, such as moral hazard, might also limit profit opportunities (see, e.g., Duffie and Singleton (2003)).

implicit from the constraint in (8). In Table 7, we report calculations of $\omega_{Aaa}(N, \lambda_i, \rho)$ based on different values for $N$, our estimates of $\lambda_i$ for various credit ratings and different assumptions about default time correlations. As was evident in Figure 3, the calculations show that $\omega_{Aaa}(N, \lambda_i, \rho)$ tends to decline with $N$ and to rise with $\lambda_i$. They also show that $\omega_{Aaa}(N, \lambda_i, \rho)$ rises with default correlation $\rho$ (systematic risk adds to risk). Figure 4 illustrates this in more detail. Given $\lambda_i$, a higher $\rho$ increases the required over-collateralisation ratio at every value of $N$. Moreover, as $N$ increases, the required over-collateralisation ratio becomes relatively larger for high versus small values of $\rho$.

## 4.2 Linear Pricing

We now investigate whether our measure of risk can explain credit spreads. In particular, we attempt to explain average spreads for each of four rating classes — namely, Aa, A, Baa and Ba — but doing so only relative to Aaa spreads. We will assume that Aaa spreads themselves are explained adequately by tax, liquidity and systematic risk. In our calculations, we match default intensities to credit ratings so that we can measure risk in portfolios for each rating. In measuring risk, we also allow the size of the portfolio and the default correlations within the portfolio to vary. Specifically, we examine whether

$$\frac{S_{P(i)}(t) - S_{Aaa}(t)}{\omega_{Aaa}(N_i, \lambda_i, \rho_i)} = \frac{S_{P(j)}(t) - S_{Aaa}(t)}{\omega_{Aaa}(N_j, \lambda_j, \rho_j)} \equiv J \tag{9}$$

holds for credit portfolios $P(i)$ and $P(j)$, $i \neq j$, where $S_{P(i)}(t)$ is the spread on portfolio $P(i)$ and $S_{Aaa}(t)$ is the spread on a benchmark Aaa-rated portfolio. In (9), we specify spreads as differentials relative to Aaa spreads, using the typical rating of the senior tranche in a CDO as a measure of risk in the benchmark portfolio. This also has the advantage of netting out the possible effects of taxes and any liquidity risk premia in corporate bonds (assuming these premia are constant across ratings).

Ideally, we would like to test (9) by confronting it with a large number of cross-sectional observations on spreads and estimates of physical default intensities, along with estimates of $N$ and $\rho$. Estimates of the average values of $\lambda_i$ across ratings classes are provided in Table 2. The analysis above helped shed some light on the typical size of corporate bond portfolios; namely, $N$ is likely around 100-200, but values as low as 50 or as great as 500 are also possible. Finally, a consensus value of $\rho$ has not yet emerged in the profession, though estimates between 0 and 0.3 have been obtained in several studies. In the face of uncertainty regarding values of $N$ and $\rho$, we can assess whether (9) matches the stylized facts on spreads and physical intensities across rating classes for a range of plausible values of $N$ and $\rho$. Table 8 reports our calculations of risk premia (where we adjust expected losses for average recovery rates). Figures 5a-c plot the differential risk premia against our calculations of risk in terms of $\omega_{Aaa}(N, \lambda_i, \rho)$ for different values of $N$ and $\rho$. (9) implies that the observations will fall on a straight line drawn from the origin. In Figures 5a-c, it appears that an assumed asset correlation of $\rho = 0.3$, with $N = 50$ or $N = 100$, results in the straightest lines.

Table 8 confirms this notion. The table reports $J$ in (9) for different rating classes and different assumptions about portfolio size and correlation. For each set of assumptions, the table also reports the mean and standard deviation of the ratios. An assumed asset correlation of 0.30 and portfolio size of 50 results in the lowest standard deviation. However, the standard deviation for an assumed asset correlation of 0.30 and portfolio size of 100 is not very different. The estimated market price of risk is 136 basis points for the smaller portfolio and 150 basis points for the bigger one. A correlation of 0.30 and corporate bond portfolios of 50 and 100 names in size are not implausible. Hence, these calculations suggest that (9) fits the stylized facts about average spreads and default intensities across ratings.

# 5 Conclusions

We have argued that feasible portfolios in credit markets are not nearly large enough for investors to diversify away idiosyncratic jump risk. A good candidate for the portfolio of the marginal investor in credit markets is an arbitrage CDO. This class of instruments has been among the fastest growing segments of the market, and CDO managers have a strong incentive to diversify. Nonetheless, actual arbitrage CDOs typically make do with no more than 250 names in their collateral pool, even for synthetic structures. With just this number of names and with the skewness in return distributions induced by the possibility of default, any such portfolio would not be diversified. In other words, idiosyncratic jump risk — or what risk managers often call the risk of unexpected losses — would remain significant.

If idiosyncratic jump risk is the risk that investors in credit markets worry about, how would we measure it? We believe that market participants are implicitly already converging on a common risk measure. The subordination structures of CDOs rely, essentially, on a VaR measure — more specifically, VaR at the confidence level consistent with the survival probability of Aaa-rated instruments. For all practical purposes, a confidence level this high would make the VaR risk measure a coherent one. The specific risk measure we propose is the ratio of this particular VaR to the size of the portfolio in question. We call this risk measure "omega". This measure takes account of portfolio size in the right way: risk declines as the portfolio gets larger. The risk measure also captures the effect of default correlations on the risk of unexpected losses.

If indeed omega was the risk measure driving credit markets, then it should explain credit spreads. To test the plausibility of this idea, we conduct a pricing exercise by calculating omega for portfolios of different sizes, different credit ratings and different default correlations. We also compute (for investors who hold these portfolios to maturity) expected excess returns based on average spreads and expected losses from default, using portfolios of Aaa-rated bonds as a benchmark. We find that for portfolio sizes typical for arbitrage CDOs and for reasonable assumptions about default correlations, the relationship between omega and differential excess returns is positive and approximately linear. We take this result as suggestive evidence that the risk of unexpected losses as measured by omega adequately explains corporate spreads.

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Table 1

Average Corporate Bond Spreads

(in basis points)

| Rating | Duration (years) | | |
|---|---|---|---|
| | 2 | 5 | 7 |
| United States | | | |
| Aaa | 47.9 | 70.1 | 74.0 |
| Aa | 56.5 | 80.7 | 86.4 |
| A | 83.8 | 108.1 | 114.0 |
| Baa | 161.3 | 181.5 | 176.1 |
| Ba | 398.3 | 338.5 | 317.0 |
| Europe | | | |
| Aaa | 24.3 | 33.7 | 37.0 |
| Aa | 34.7 | 49.5 | 55.1 |
| A | 58.0 | 83.1 | 89.9 |
| Baa | 127.5 | 132.9 | 157.9 |

Notes: Sample averages of Merrill Lynch's corporate bond option-adjusted spread indices over the periods January 1997 - July 2004 (United States) and January 1999 - July 2004 (Europe). "Duration" is the average duration (rounded to nearest integer) of bonds in the indices as of July 2004 (Merrill Lynch provides index spreads by maturity buckets); durations of 2, 5 and 7 years correspond to maturity buckets 1-3, 5-7 and 7-10 years, respectively.

Table 2
Default Probabilities, Expected Loss and Spreads
5-Year Horizon
(in basis points, except for spread ratio)

| Rating | 5-year Default Prob. | Intensity | Exp. Loss | Spread | Spread Ratio | Spread Difference | Excess Return vs. Aaa |
|---|---|---|---|---|---|---|---|
| | | | | United States | | | |
| Aaa | 0.5 | 0.1 | 0.1 | 70.1 | 625.4 | 70 | - |
| Aa | 8.0 | 1.6 | 0.9 | 80.7 | 55.4 | 79.8 | 9.8 |
| A | 52.9 | 10.6 | 6.2 | 108.1 | 13.2 | 101.9 | 41.9 |
| Baa | 340.4 | 69.3 | 40.1 | 181.5 | 4.1 | 141.4 | 71.4 |
| Ba | 1255.3 | 268.3 | 147.9 | 338.5 | 2.2 | 190.6 | 120.6 |
| | | | | Europe | | | |
| Aaa | 0.4 | 0.1 | 0.1 | 33.7 | 209.4 | 33.6 | - |
| Aa | 5.0 | 1.0 | 0.6 | 49.5 | 35.1 | 48.9 | 15.3 |
| A | 58.3 | 11.7 | 6.9 | 83.1 | 6.7 | 76.2 | 42.6 |
| Baa | 454.1 | 92.9 | 53.5 | 132.9 | 1.6 | 79.4 | 45.8 |

Notes: See the text for details on computations. Sources: Merrill Lynch, Moody's.

Table 3
Expected Excess Returns, Volatility and the Sharpe Ratio

| | Duration (years) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | | | 5 | | | 7 | | |
| Rating | Excess Return | Vol | Sharpe Ratio | Excess Return | Vol | Sharpe Ratio | Excess Return | Vol | Sharpe Ratio |
| United States | | | | | | | | | |
| Aaa | 1.07 | 1.71 | 0.63 | 2.01 | 5.03 | 0.40 | 2.33 | 6.11 | 0.38 |
| Aa | 1.15 | 1.73 | 0.66 | 2.10 | 4.92 | 0.43 | 2.44 | 5.96 | 0.41 |
| A | 1.37 | 1.65 | 0.83 | 2.32 | 4.75 | 0.49 | 2.67 | 5.81 | 0.46 |
| Baa | 1.81 | 2.15 | 0.84 | 2.72 | 4.72 | 0.58 | 2.95 | 5.62 | 0.52 |
| Ba | 3.10 | 5.80 | 0.53 | 3.21 | 5.64 | 0.57 | 3.28 | 7.19 | 0.46 |
| Europe | | | | | | | | | |
| Aaa | 0.58 | 1.31 | 0.45 | 1.34 | 3.35 | 0.40 | 1.70 | 4.22 | 0.40 |
| Aa | 0.68 | 1.30 | 0.52 | 1.49 | 3.39 | 0.44 | 1.87 | 4.20 | 0.45 |
| A | 0.85 | 1.43 | 0.60 | 1.76 | 3.48 | 0.51 | 2.16 | 4.05 | 0.53 |
| Baa | 1.08 | 1.76 | 0.61 | 1.80 | 3.28 | 0.55 | 2.37 | 6.00 | 0.40 |

Table 4
Composition of Largest US Open-end Corporate Bond Funds

| Fund Name | Size | No. of Issues | No. of Issuers | % in Corporates | Duration | Rating Mode |
|---|---|---|---|---|---|---|
| Investment Grade | | | | | | |
| Vanguard IT Investment | 4339 | 533 | 385 | 81.35 | 4.9 | Baa |
| Janus Flexible Income | 1166 | 265 | 162 | 72.14 | 4.6 | Baa |
| Aim Income | 807 | 444 | 204 | 76.67 | 3.8 | Aaa |
| Columbia Income | 547 | 254 | 191 | 93.04 | 4.9 | Baa |
| Strong Corporate | 540 | 203 | 130 | 94.26 | 6.4 | Baa |
| Total (unique) | - | 1501 | 685 | - | - | - |
| High Yield | | | | | | |
| American High-Income Trust | 8991 | 780 | 363 | 90.84 | 4.6 | B |
| Vanguard High Yield | 8855 | 403 | 250 | 94.3 | 3.6 | B |
| PIMCO High Yield | 6700 | 495 | 219 | 81.66 | 4.2 | B |
| MainStay High Yield | 4710 | 463 | 261 | 81.97 | 4.4 | B |
| Fidelity Capital and Income | 4207 | 508 | 253 | 86.99 | 4.7 | B |
| Total (unique) | - | 1932 | 831 | - | - | - |

Notes: Figures as of 24 August 2004. This table reports the top five investment grade and high yield funds from a pool of open-end funds that invest at least 70 per cent on corporate bonds, are domiciled in the United States and have at least USD 300 million worth of assets (fund size in millions of USD). Duration is computed as the average across securities. "Ratings Mode" is the mode of the ratings distribution of fund securities based on the Bloomberg composite ratings. Source: Bloomberg.

Table 5
Composition of Collateral Pools
in Cash Arbitrage CDOs

| | Mean | Max | 80th Percentile | 20th Percentile | Min | Standard Deviation |
|---|---|---|---|---|---|---|
| Investment Grade CDOs | | | | | | |
| Diversity Score | 50.69 | 64 | 56.5 | 45 | 34.1 | 7.26 |
| Average Rating | Ba1 | Baa2 | Baa3 | Ba2 | Ba3 | - |
| Average Maturity | 5.78 | 8.26 | 6.71 | 4.63 | 3.47 | 1.14 |
| High Yield CDOs | | | | | | |
| Diversity Score | 42.44 | 71.8 | 55.7 | 30.88 | 1 | 14.67 |
| Average Rating | B3 | Ba1 | B2 | Caa1 | Ca-C | - |
| Average Maturity | 4.82 | 7.99 | 5.78 | 3.91 | 1.12 | 1.12 |

Notes: Statistics are cross-sectional averages across CDO deals based on underlying collateral. "Average Rating" is converted from the Adjusted Moody's Rating Factor. "Average Maturity" is a weighted average, in years. Source: Moody's.

Table 6
Tranche Sizes in Cash Arbitrage CDOs

| Market Averages | | | | |
|---|---|---|---|---|
| | Investment Grade | | High Yield | |
| Tranche | Size ($MM) | Portion (%) | Size ($MM) | Portion (%) |
| Senior | 383.08 | 81.2 | 239.50 | 66.6 |
| Mezzanine | 52.97 | 11.2 | 69.47 | 19.3 |
| Equity | 35.87 | 7.6 | 50.44 | 14.0 |
| Total | 471.92 | 100 | 359.41 | 100 |

| Example: Diamond Investment Grade CDO, Ltd. I | | |
|---|---|---|
| Tranche | Size ($MM) | Portion (%) |
| Senior | 415 | 83 |
| Mezzanine | 65 | 13 |
| Equity | 20 | 4 |
| Total | 500 | 100 |

Notes: "Size" and "Portion" are averages, at issuance. Market-wide data covers the period 1987-2004. Source: JPMorgan Chase.

Table 7
Value-at-Risk
Aaa-rated Confidence Level, 5-Year Horizon

| Collateral Rating | Portfolio Size | | | |
|---|---|---|---|---|
| | 50 | 100 | 500 | 1000 |
| Correlation = 0 | | | | |
| Aa | 4 | 3 | 1 | 0.6 |
| A | 8 | 5 | 2.2 | 1.6 |
| Baa | 16 | 12 | 7 | 5.9 |
| Ba | 34 | 27 | 18.6 | 16.8 |
| Correlation = 0.1 | | | | |
| Aa | 6 | 4 | 1.8 | 1.6 |
| A | 12 | 9 | 6 | 5.4 |
| Baa | 30 | 25 | 20.2 | 19.3 |
| Ba | 56 | 50 | 44.6 | 43.4 |
| Correlation = 0.3 | | | | |
| Aa | 8 | 7 | 4.6 | 4.2 |
| A | 22 | 19 | 15.4 | 14.7 |
| Baa | 52 | 48 | 43.6 | 42.6 |
| Ba | 82 | 79 | 74.2 | 73.5 |

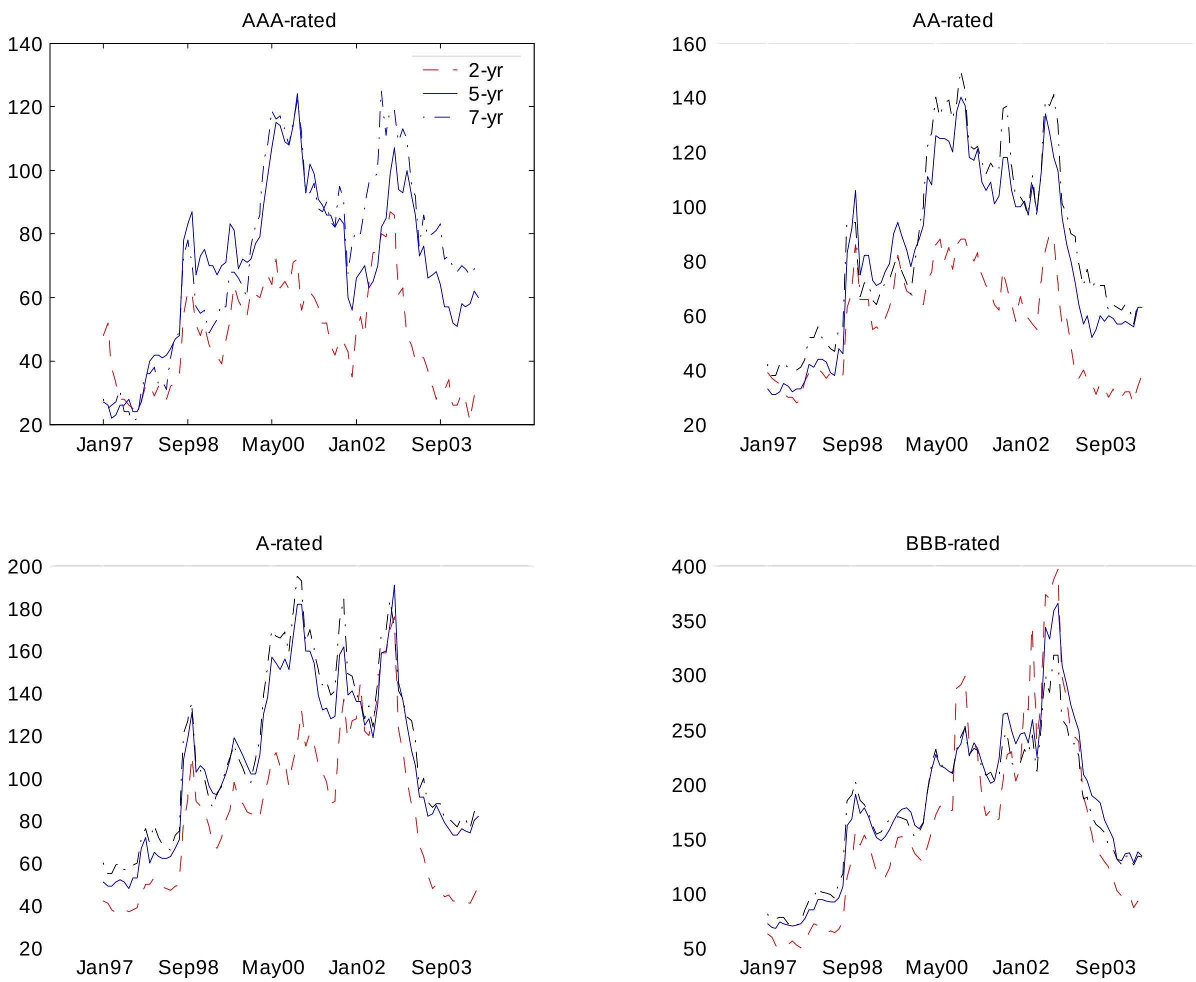
Notes: As a percentage of portfolio size.

Table 8
Excess Returns versus Value-at-Risk
Aaa-rated Confidence Level, 5-Year Horizon

| Rating | Portfolio Size | | | |
|---|---|---|---|---|
| | 50 | 100 | 500 | 1000 |
| Correlation = 0 | | | | |
| Aa | 236 | 315 | 945 | 1575 |
| A | 397 | 635 | 1443 | 1984 |
| Baa | 450 | 600 | 1029 | 1221 |
| Ba | 340 | 428 | 622 | 688 |
| Mean | 356 | 495 | 1010 | 1367 |
| Standard Deviation | 92 | 150 | 338 | 550 |
| Correlation = 0.1 | | | | |
| Aa | 157 | 236 | 525 | 591 |
| A | 265 | 353 | 529 | 588 |
| Baa | 240 | 288 | 357 | 373 |
| Ba | 207 | 231 | 259 | 266 |
| Mean | 217 | 277 | 418 | 455 |
| Standard Deviation | 47 | 57 | 133 | 162 |
| Correlation = 0.3 | | | | |
| Aa | 118 | 135 | 205 | 225 |
| A | 144 | 167 | 206 | 216 |
| Baa | 139 | 150 | 165 | 169 |
| Ba | 141 | 146 | 156 | 157 |
| Mean | 136 | 150 | 183 | 192 |
| Standard Deviation | 12 | 13 | 26 | 34 |

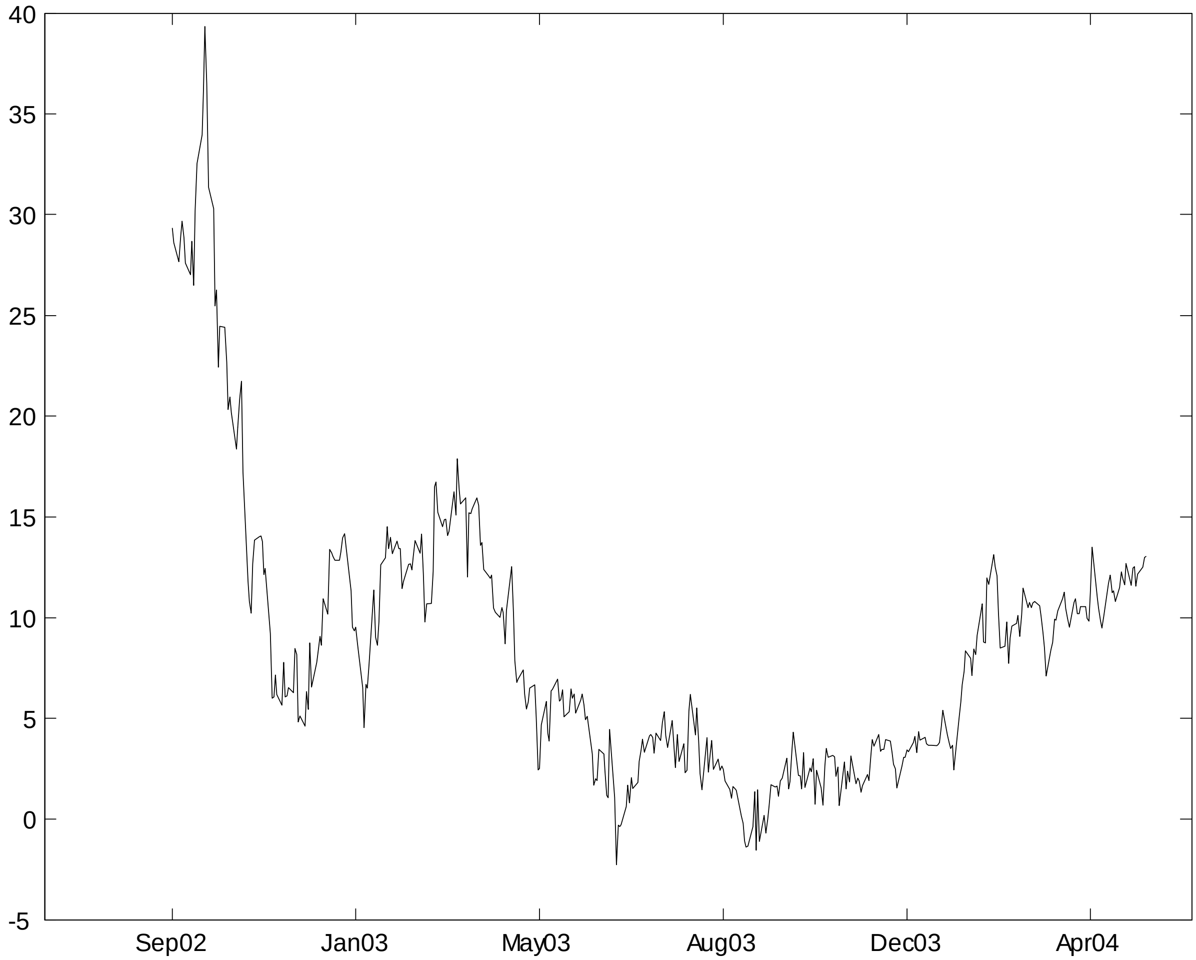
Notes: Excess return is calculated as spread minus AAA-spread, after adjusting for expected loss (in basis points).

Figure 1
US Corporate Bond Spreads




Notes: Option-adjusted spread indices on US corporate bonds from Merrill Lynch (in basis points).

Figure 2
European Credit Default Swap Basis




Notes: Default swap basis is defined as CDS spread minus asset swap spread (in basis points). Calculated based on entities in Trac-x European investment grade index. Source: JP Morgan Chase.

Figure 3
Required Over-collateralisation Ratio in an Arbitrage CDO

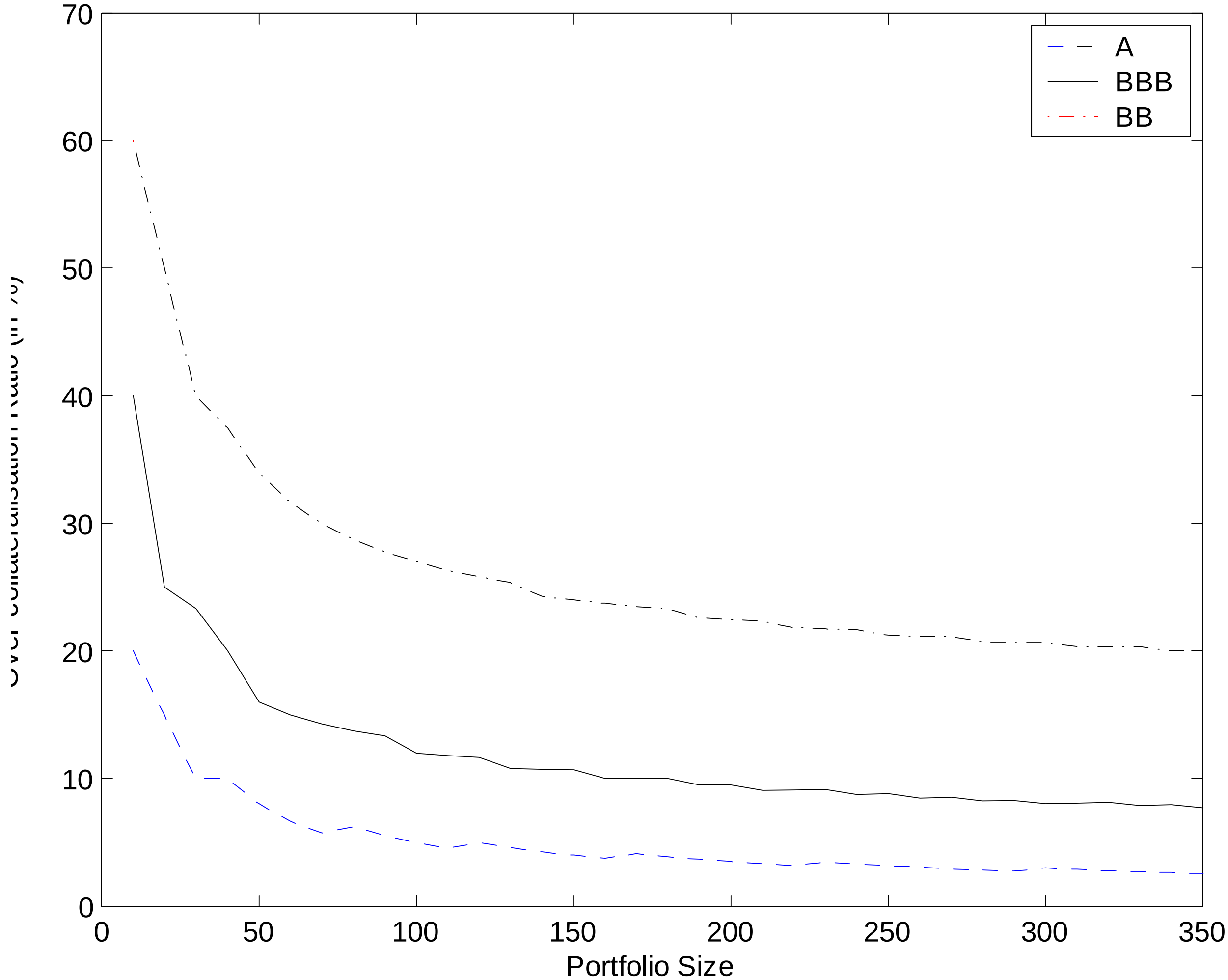


Figure 4
Required Over-collateralisation Ratio in an Arbitrage CDO:
Effects of Correlation

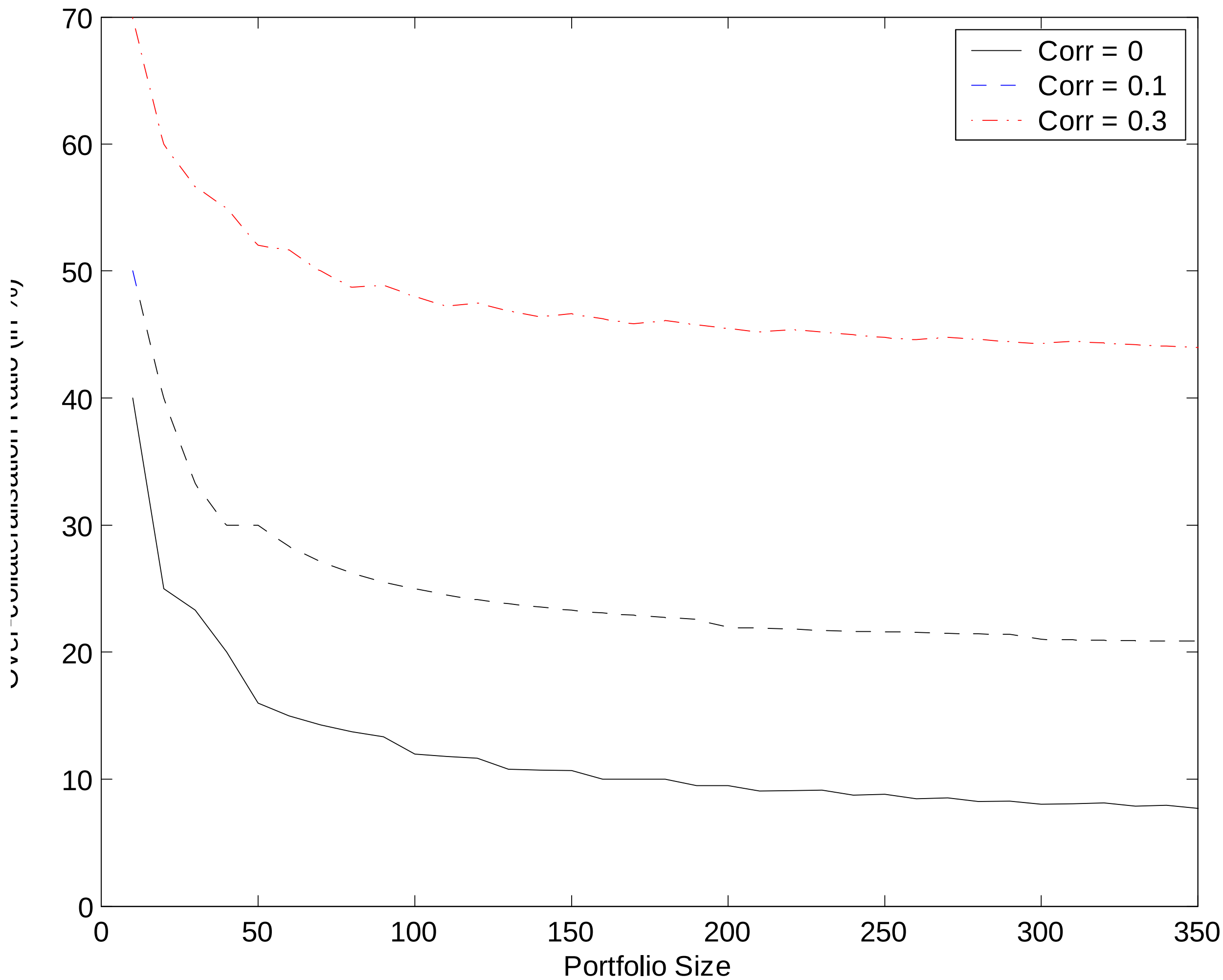


Figure 5a
Excess Returns vs. VaR
Aaa-rated Confidence Level, 5-Year Horizon
Correlation = 0

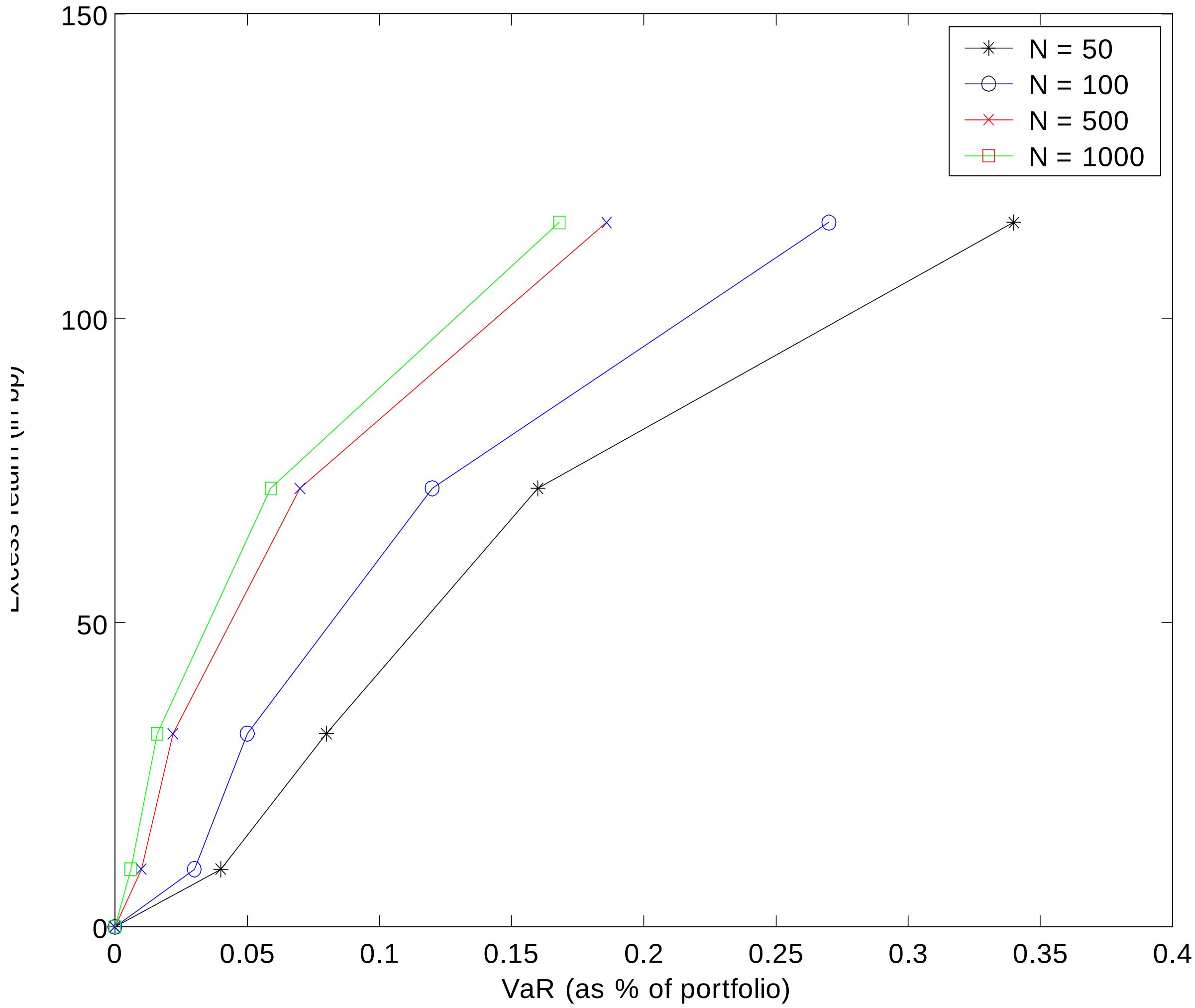


Figure 5b
Excess Returns vs. VaR
Aaa-rated Confidence Level, 5-Year Horizon
Correlation = 0.1

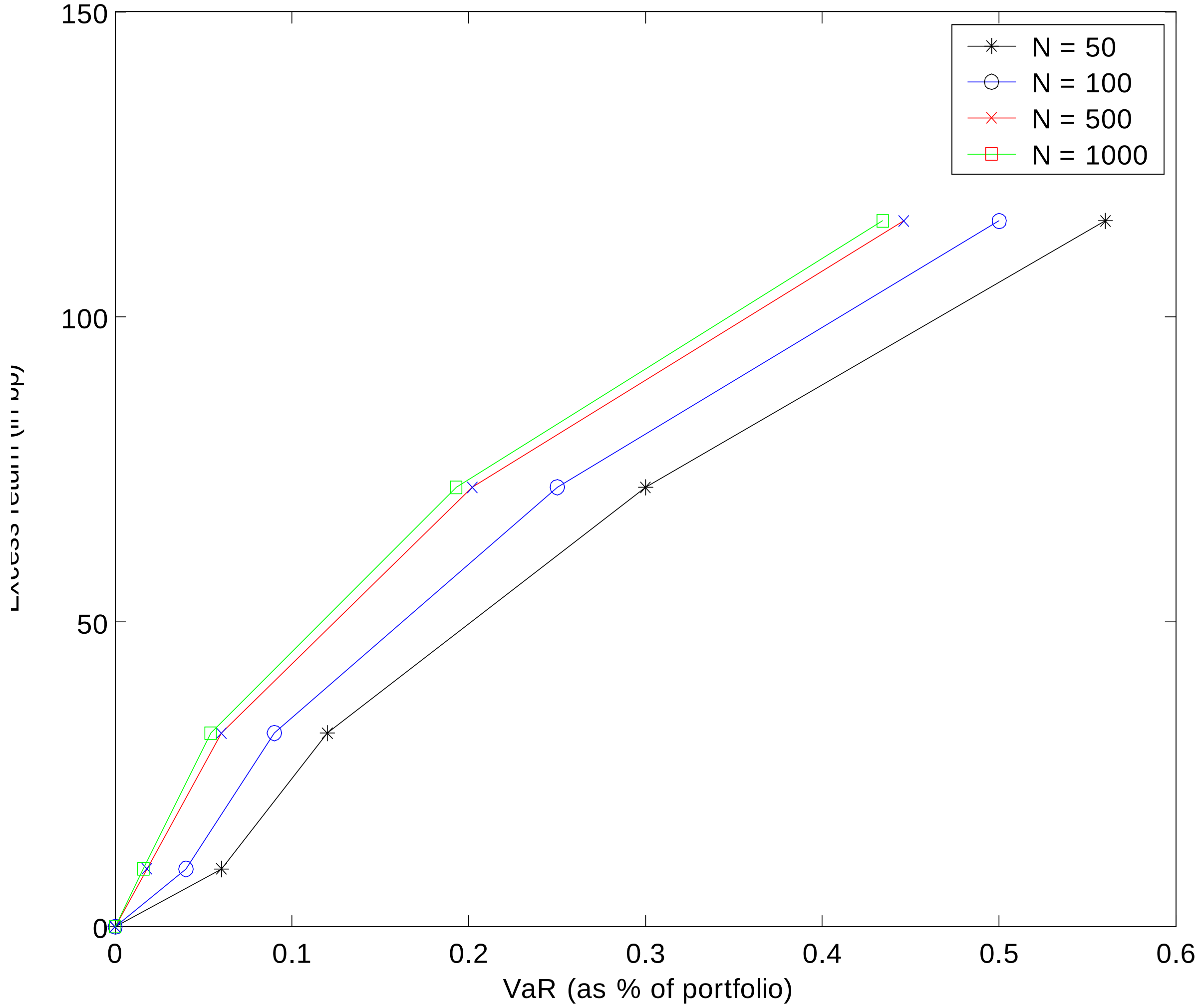


Figure 5c
Excess Returns vs. VaR
Aaa-rated Confidence Level, 5-Year Horizon
Correlation = 0.3