A Model of Swap Spreads and Corporate Bond Yields[1]

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Abstract

We propose a joint six-factor affine model for Treasury bonds, corporate bonds, and swap rates with the purpose of estimating credit and liquidity components in swap spreads. In the pricing of Treasury bonds we allow for a convenience yield unique to these bonds. Corporate bonds are priced using an intensity-based, affine framework taking into account rating migrations as in Lando (1998). We follow Dufresne and Solnik (2001) in the pricing of interest rate swaps by assuming that swap contracts are free of counterparty risk and discount the floating and fixed leg of the swap contract with the riskless rate in order to find the fair swap rate. Although the swap contract is default free, the floating rate leg in the swap is tied to the credit-risky LIBOR and the swap rate reflects this credit risk. In the empirical work we estimate our model using weekly US data for government bonds, interest rate swaps and corporate bonds rated AAA, AA, A, and BBB for the period 1996-2003. We estimate three components of swap spreads: 1) a credit risk component that is due to the credit risk in LIBOR, 2) a swap factor that is unique to the swap market, and 3) a convenience yield component of Treasury securities. We find that the credit risk component is important and increasing with the maturity of the swap, but the commonly used difference between LIBOR and GC Repo rates is a poor proxy for this component. We relate the swap factor to hedging of interest rate risk in MBS markets and find it to be increasing with maturity. We find that the Treasury component is decreasing with maturity and that it is less volatile than suggested by existing proxies.

1 Introduction

Interest rate swaps and Treasury securities are the primary instruments for hedging interest rate risk in the MBS and corporate bond markets but the large widening of swap spreads - the difference between swap rates and comparable Treasury yields - in the fall of 1998 clearly revealed that there are important differences between the two markets. The ability to accurately hedge interest rate risk critically depends on understanding these differences. The swap spread cannot be explained solely by the fact that the floating rate payment in the swap is linked to a LIBOR rate which contains at least some compensation for credit risk. An additional contribution to the spread comes from a convenience yield of Treasury securities arising from among other things specialness effects in repo markets. Some previous studies have focused on understanding the influence on swap spreads from these two components by incorporating observable proxies for credit risk and liquidity in a time series framework. Others set up pricing models for riskless rates, swap rates and yields on Treasury securities but still rely on the observable proxies to identify credit risk and a convenience yield in the models. The reliability of these results clearly depends on the appropriateness of these proxies.

This paper proposes a joint pricing model for Treasury securities, corporate bonds and swap rates using six latent factors. Two factors are used in the model of the government yield curve, one factor is used in modeling the convenience yield in Treasuries, two factors are used in the credit risk component in corporate bonds, and one is a factor unique to the swap market. After obtaining a good joint fit of Treasury yields, corporate yields and swap rates, we extract information about several key quantities which are identified in our model and compare them to historical evolutions of variables that are often used as proxies. We stress that this comparison is only possible because we extend the pricing model to include corporate bonds. Our key findings are as follows:

1. LIBOR - GC repo rates are much too volatile to serve as a proxy for the short term AA-credit spread. Interpreting this spread as a pure credit spread is inconsistent with the pricing implications for corporate bonds.

2. To fit the markets simultaneously, we need a factor which is specific to the swap market, similar to the idiosyncratic swap factor used in Reinhart and Sack (2002). The presence of this factor implies that

the assumption of *homogeneous credit quality* (defined below) in the LIBOR and AA corporate markets cannot be maintained. After 2000, this factor has a strong correlation with hedging activity in the MBS market.

3. To fit markets simultaneously, we also need a factor specific to Treasury securities - a factor commonly interpreted as a convenience yield to owning Treasuries. Various proxies for this factor have been proposed in the literature. We find that the estimated Treasury factor is less volatile than the GC repo - T-Bill spread. We also compare the factor with the difference between yields on Refcorp strips and Treasury bonds and find that this difference is inappropriate as a proxy of the Treasury factor in the short end of the yield curve.

Our model builds upon and extends a number of previous models and empirical studies. In Duffie and Singleton (1997) the 6-month LIBOR rate is based on an adjusted short rate process R which includes the Treasury rate, an adjustment for liquidity differences in Treasury and swap markets, and a loss adjusted default rate. By simultaneously using R to discount the cash-flows of the swap and for determining the floating rate payments of the swap, the fair swap rates depend only on R and not on the contributions from the individual components to R. In their subsequent analysis, the swap rates are therefore regressed on proxies for liquidity and credit risk, but the components are not included separately in the pricing model[1].

We follow Collin-Dufresne and Solnik (2001) and find the fair swap rate by pricing the cash flows of the swap separately using an (estimated) riskless rate (instead of using the refreshed LIBOR rate as in Duffie and Singleton (1997)). This reflects the fact that counterparty risk on plain vanilla interest rate swap is typically eliminated through posting collateral and netting agreements. As noted in Collin-Dufresne and Solnik (2001), future paths of the LIBOR-rate are critical in determining the swap rates. Large future LIBOR rates will imply higher swap rates. The viewpoint in our paper is that only by including corporate bond rates can we reasonably hope to separate out from LIBOR that part which is due to credit risk. The credit risk is reflected in part in the corporate AA-curve, but this in turn is affected by adjacent

[1]Duffie and Singleton do discuss a specification which separates out the riskless short rate and a combined liquidity and credit risk adjustment, but this specification is not estimated in their paper.

curves since a bond currently rated AA is affected by default risk in adjacent rating-categories. Our joint modeling of corporate curves and the swap curve therefore gives a much more detailed view on the future path of LIBOR rates and on the AA corporate curve than the one used in Collin-Dufresne and Solnik (2001). Also, we focus on explaining factors influencing swap spreads while Collin-Dufresne and Solnik (2001) focus on the difference between the swap curve and the AA corporate curve.

Both Duffie and Singleton (1997) and Collin-Dufresne and Solnik (2001) assume that the 6-month AA corporate rate and 6-month LIBOR are the same - an assumption Duffie and Singleton (1997) refer to as homogeneous credit quality. We cannot maintain this assumption and obtain a simultaneous fit of swap and corporate bond rates.

Our approach is similar to that of Liu, Longstaff, and Mandell (2004), who use a five-factor model using three factors to model Treasury yields, one factor to model the 'liquidity' (i.e. what we refer to as the convenience yield) of Treasury securities and one factor for default risk. Their identification of the credit risk and the liquidity component in swap spreads relies critically on the use of 3-month GC repo rates as a short term riskless rate and 3-month LIBOR as a credit-risky rate. Their default factor is in fact equal by definition to the difference between 3-month LIBOR and 3-month GC repo rates, an assumption used also (for 1-month rates) by He (2001). By including the information of corporate bonds in our study we do not need to rely on short-term interest rate spreads as proxies for credit risk and Treasury components and we show that this strongly alters conclusions about the size and time series behavior of these components.

Most of the various proxies that we discuss in this paper, can be found in the multivariate time series model of Reinhart and Sack (2002) who specify a multivariate time series model for 10 year swap rates, off-and on-the run Treasury rates, Refcorp rates and AA corporate rates. However, their model does not contain any pricing model or full term structure modeling of the relevant rates.

Grinblatt (2001) takes a different approach and views swap rate as the riskless rate and the spread between government and swap rates as a liquidity spread. The argument presented in Grinblatt (2001) relies on AA refreshed credit as being virtually riskless. While it is true that historical default experience for AA issuers over a three month or six month period is extremely low, we do find a credit risk component in swap spreads.

The rating-based approach explicitly incorporates different dynamics for bonds of different rating categories. This is consistent with empirical evidence in Duffee (1999) who finds that the dynamics of the hazard rates depends on the rating category. Hence viewing the firm hazard rate as a diffusion and thinking of rating deteriorations as being represented by high levels of this diffusion seems less plausible. We could in theory model the yields for different rating categories by adding positive valued processes for lower categories, but unless we include a migration component as well we cannot price bonds consistently.

Our approach also allows for a stochastic variation in spreads which can be due either to a time varying risk premium on default event risk or to changing hazard rates. Driessen (2005) finds evidence in support of the existence of an event risk premium of default, but the evidence is not conclusive. He assumes, however, in his empirical specification that a possible risk premium on default event risk is constant. If there is a time varying risk premium on default event risk, the risk premium on variations of default risk will not be estimated properly. Our specification allows for time varying event risk premium and therefore also for a more reliable estimation of the risk premium on variations in default risk.

The outline of the paper is as follows: In Section 2 we describe the structure of our model. The explicit pricing formulas are relegated to an appendix. Section 3 describes the US market data, that we use, and Section 4 explains our estimation methodology. In Section 5 we report our parameter estimates along with residuals from the estimation, and we elaborate on our main findings, as outlined above. Section 6 concludes.

2 The Model

Our model of Treasury bonds is an affine short rate model with a liquidity component, and we use an intensity-based, affine framework for corporate bonds and swaps as introduced in Duffie and Singleton (1997,1999) and Lando (1994, 1998). Since our pricing of corporate bonds includes rating information we also use the affine, rating based setting introduced in Lando (1994, 1998).

We use a six-factor models based on independent translated CIR processes. More precisely, we assume that the latent state vector X consists of 6 independent diffusion processes with an affine drift and volatility structure,

$$\begin{aligned} X_t &= (X_{1t}, ..., X_{6t})' \\ dX_{it} &= k_i(X_{it} - \theta_i)dt + \sqrt{\alpha_i + \beta_i X_{it}} dW_i^P, i = 1, ..., 6, \end{aligned}$$

where the Brownian motions $W_1^P, ..., W_6^P$ are independent. This specification nests the Vasicek ($\beta = 0$) and CIR($\alpha = 0$) processes as special cases. We assume that the market price of risk for factor i is proportional to its standard deviation and normalize the mean of X_i under Q to zero for identification purposes, so the processes under Q are given by

$$dX_{it} = k_i^* X_{it} dt + \sqrt{\alpha_i + \beta_i X_{it}} dW_i^Q,$$

where

$$\begin{aligned} k_i^* &= k_i - \lambda_i \beta_i \\ \lambda_i &= -\frac{k_i \theta_i}{\alpha_i}. \end{aligned}$$

From the state vector we now define the short rate processes, intensities and liquidity adjustments needed to jointly price the Treasury and corporate bonds and the swap contracts.

We work in an arbitrage-free model with a riskless short rate and this rate r is given as a three-factor process,

$$r(X) = a + X_1 + X_2 + (e + X_5), \tag{1}$$

where the first two factors X_1 and X_2 are the factors governing the Treasury short rate while the last factor X_5 is a Treasury factor which distinguishes the Treasury rate from the riskless rate. Government bonds are assumed to have a lower yield than riskless bonds due to their convenience yield arising from repo specialness and their special status as investment vehicle for institutional investors (see Duffie (1996)). The constant a is the Q-mean of the government short rate while e is the Q-mean of the convenience yield. Consequently, the Treasury short rate process is given as

$$r^g(X) = a + X_1 + X_2. \tag{2}$$

The Treasury factor is positive and in the empirical work we restrict the parameters such that $e + X_5$ is positive. From this affine specification, prices of government bonds are given as

$$P^g(t, T) = \exp(A^g(T - t) + \mathbf{B^g}(\mathbf{T} - \mathbf{t})' X_t),$$

where A^g and $\mathbf{B^g}$ can be found in appendix B.2.

Our model for corporate bonds prices a 'generic' bond with initial rating i by taking into account both the intensity of default for that rating category and the risk of migration to lower categories with higher default intensities. Spread levels within each rating category are stochastic, but for all rating categories they are modeled jointly by a stochastic credit spread factor.

We use the reduced form representation with fractional recovery of market value to price a corporate bond which at time t is in rating category $\eta_t = i$:

$$v^i(t,T) = E_t^Q \exp\big(- \int_t^T (r(X_u) + \lambda(X_u, \eta_u)du)\big). \tag{3}$$

where $\lambda(X, \eta)$ is the loss-adjusted default intensity when the rating-class is η. The default intensities for the different categories are assumed to have a joint factor structure

$$\lambda(X, i) = \nu_i \mu(X)$$

where ν_i's are constants, and $\mu(X)$ is a strictly positive process which ensures stochastic default intensities for each rating category and plays the role as a common factor for the different default intensities. We specify μ as

$$\mu(X) = b + X_3 + X_4 + c(X_1 + X_2).$$

Note that the process μ is allowed to depend on government rates through the constant c while the two processes X_3 and X_4 are used only in the definition of μ. Hence we have in essence a two factor model for the credit spreads across different rating categories. We now have the definition of the loss adjusted default intensity as a function of the state variable process and the rating category, so all that is left to specify before (3) can be evaluated, is the stochastic process for the rating migrations. We work with a 'conditional' Markov assumption as in Lando (1994, 1998) which means that the transition intensity from category i to category j is given as

$$a_{ij}(X_t) = \lambda_{ij} \mu(X_t)$$

where λ_{ij} is a constant for each pair $i \neq j$ and $\mu(X_t)$ is the same factor that governs credit spreads. We can collect all the conditional transition intensities and the loss adjusted default intensities into one common matrix given as

$$A_X(s) = \Lambda^\nu \mu(X_s).$$

Note that the scalar function $\mu(X_s)$ is multiplied onto every element of the (loss-adjusted) generator matrix Λ^ν. This means essentially that the intensities of rating and default activity is modulated by the process $\mu(X_s)$. The multiplicative effect has the effect of modifying the default intensity by a scalar function which captures both stochastic variation in this and possibly a compensation for event risk.[2] As shown in Lando (1994, 1998), this specification generates pricing formulas for corporate bonds in all rating categories which are sums of affine functions. Hence the price of a zero coupon corporate bond in rating class i at time t is of the form

$$v^i(t,T) = \sum_{j=1}^{K-1} c_{ij} E_t(\exp(\int_t^T d_j \mu(X_u) - r(X_u) du)) \tag{4}$$

where the constants c_{ij} and d_j are given in appendix B.3. Our model does not take into account the difference in the tax treatment between corporate bonds and Treasury securities. Grinblatt (2001) argues that 'the tax equilibrium argument is implausible because the state tax advantage does not apply to broker-dealers, tax-exempt investors like pension funds, or international investors[3] who would then arbitrage away these differences'. Elton et al. (2001) employ a 'marginal investor' tax rate argument and estimate the tax premium on corporate bonds to be significant. They do however measure the bond spreads using Treasury bonds as a benchmark. The convenience yield that we estimate for Treasury bonds easily explains a spread of similar magnitude. Longstaff, Mithal, and Neis (2005) in their analysis of the non-default component of credit spreads for corporate bonds find only weak support for a tax effect.

With the specification of the Treasury and corporate bond prices in place, we can now find swap rates. First, we need to define the 3-month LIBOR rate used to determine the floating-rate payment on the swap:

$$L(t,t+0.25) = \frac{1}{v^{LIB}(t,t+0.25)} - 1$$

[2]Since we do not observe empirical default intensities, we cannot decompose this multiplicative factor into event risk premium and variation in default risk under the empirical measure.

[3]Tony Crescenzi of BondTalk.com reports in a market Commentary on November 18, 2004, that foreign corporate bond purchases reached a record of 44 billion dollars in September 2004

where $v^{LIB}(t, t+0.25)$ is the present value of a 3-month loan in the interbank market:

$$v^{LIB}(t, t + 0.25) = E_t^Q \exp(- \int_t^{t+0.25} \lambda^{LIB}(X_s)ds). \tag{5}$$

The adjusted short rate to value this loan is given as

$$\lambda^{LIB}(X_s) = r(X_s) + \nu_{AA}\mu(X_s) + S(X_s). \tag{6}$$

There are three stochastic components in the determination of LIBOR rates. First component is the riskless rate $r(X_s)$. The second component, $\nu_{AA}\mu(X_s)$, is the loss adjusted AA-intensity of default. If these were the only two components defining LIBOR, we would be working under the assumption that the three-month LIBOR rate and the yield on a 3-month *AA* corporate bond are equal. This is an assumption typically used in the literature (Duffie and Singleton (1997), Collin-Dufresne and Solnik (2001), Liu, Longstaff, and Mandell (2004), and He (2001)). However, Duffie and Singleton (1997) note that the assumption - which they call *homogeneous LIBOR-swap market credit quality* - is nontrivial since the default scenarios, recovery rates, and liquidities of the corporate bond and swap markets may differ. The additional component $S(X_s)$, which we use, accounts for such differences and as we will discuss later this component has important consequences for the model's ability to fit swap rates. We assume that the component $S(X_s)$ that allows for differences in swap and corporate bond markets is defined by

$$S(X) = d + X_6.$$

In contrast to the other 5 factors, $S(X)$ only comes into play when pricing swaps. With the floating-rate payments on the swap in place, we proceed to value the swap, i.e. to find the fixed rate payments needed to give the contract an initial value of zero. We compute the value of the swap by taking present values separately of the fixed-and floating payments, and by discounting both sides of the swap using the riskless rate. This amount to ignoring *counterparty risk* in the swap contract - a standard assumption in recent papers[4]. From a theoretical perspective this assumption is justified in light of the small impact that counterparty default risk has on swap rates when

[4]See He (2001), Grinblatt (2001), Collin-Dufresne and Solnik (2001), and Liu, Longstaff, and Mandell (2004).

default risk of the parties to the swap are comparable as shown in Duffie and Huang (1996) and Huge and Lando (1999). From a practical perspective posting of collateral and netting agreements reduce - if not eliminate - counterparty risk. Bomfim (2002) shows that even under times of market distress there is no significant role for counterparty risk in the determination of swap rates.

With these assumptions we can value the swap rates in closed form. The swap data in the empirical section are interest rate swaps where fixed is paid semi-annually while floating is paid quarterly. We consider an interest rate swap contract with maturity $T-t$, where $T-t$ is an integer number of years. Defining $n = 4(T-t)$ as the number of floating rate payments at dates $t_1, ..., t_n$ and $F(t,T)$ as the $T-t$-year swap rate, the three-month LIBOR

$$L(t_{i-1}, t_i)$$

is paid at time $t_i, i = 1, ..., n$ while the fixed-rate payments

$$\frac{F(t,T)}{2}.$$

The resulting formula for the swap rate is

$$F(t,T) = \frac{2\sum_{i=1}^{n}(e^{A^s(t_{i-1}-t)+\mathbf{B^s}(t_{i-1}-t)'X_t} - P(t,t_i))}{\sum_{i=1}^{\frac{n}{2}} P(t,t_{2i})},$$

where the functions A^s and $\mathbf{B^s}$ are found in the appendix.

3 Data Description

Data consist of Treasury yields, swap rates, and corporate yields for the rating categories AAA, AA, A, and BBB on a weekly basis from 1996 to 2003. The rates obtained from Bloomberg are from the US market and covering the period from December 20, 1996, to February 14, 2003. In total 322 observations for each time series. The rates reported are closing rates on Friday.

Treasury rates are zero coupon yields and covers the maturities 0.5, 1, 2, 3, 4, 5, 6, and 7 years[5].

[5]For a review of Bloomberg's estimation methodology see OTS (2002).

Swap rates are for swaps with a semi-annually fixed rate versus 3-month LIBOR and are means of the bid and ask rates from major swap dealers' quoted rates. Data covers the maturities 2, 3, 4, 5, and 7 years. In addition to the swap data, 3-month LIBOR is used in estimation.

Corporate rates are zero coupon yields obtained from Bloomberg's Fair Market Yield Curves (FMYC) for banks/financial institutions[6] for the investment grade categories AAA, AA, A, and BBB. Corporate bond data cover the maturities 1,2, 3, 4, 5, 6, and 7 years. Yield curves for the rating category BBB are missing in the period May 5, 2000, to January 11, 2002.

Figure 1 shows the average yield curves for the period December 20, 1996, to February 14, 2003. Not surprisingly, the swap curve is well above the Treasury curve. However, the swap curve is below the AA curve and the average spread between AA yields and swap rates increases with maturity. The average 2-year spread is 14.8 basis points increasing to 34.8 at a maturity of 7 years.

[Figure 1 about here.]

In Figure 2 the 5-year AA, BBB and government yields and the swap rate from 1996 to 2003 are graphed. The 5-year BBB yield exhibits a peculiar "jump" not present in the other time series just before the missing period starts. This "jump" is present in the BBB time series for other maturities as well and indicates mispricing. The four observations for the BBB curve for the "jump" dates, from April 7, 2000 to April 28, 2000, are therefore removed. This expands the period where no BBB curves are available to the period from April 7, 2000, to January 11, 2002.

[Figure 2 about here.]

The spreads between the AA par yield and swap rate for maturities 2, 4, and 7 years from 1996 to 2003 are graphed in Figure 3. It is seen that all three spreads vary substantially and they are roughly between 0 and 20 basis points before year 2000 while increasing steadily the last three years. However, whether the spreads are small or large they largely maintain their

[6]For more information see Doolin and Vogel (1998)

order: the 7 year spread is larger than the 4 year spread, which again is larger than the 2 year spread. This indicates that the spread between the AA yield and swap rate on a specific maturity shows significant variation across time, but the gradual widening with maturity between the curves remains stable.

[Figure 3 about here.]

4 Estimation Methodology

Similar to Duffee (1999) and Driessen (2005) we estimate the model using both the cross-sectional and time-series properties of the observed yields by use of the extended Kalman filter.

Each week we observe 42 yields (we return to missing observations later):

- 8 government yields
- 7 AAA corporate yields
- 7 AA corporate yields
- 7 A corporate yields
- 7 BBB corporate yields
- 1 LIBOR rate
- 5 swap rates

We recall that $X_t = (X_{t1}, ..., X_{t6})'$ where $X_1, ..., X_6$ are 6 independent affine processes. Suppressing the dependence on the parameters the measurement and transition equation in the Kalman filter recursions are[7]

$$y_t = A_t + B_t X_t + \epsilon_t, \qquad \epsilon_t \sim N(0, H_t) \tag{7}$$

$$X_t = C_t + D_t X_{t-1} + \eta_t, \qquad \eta_t \sim N(0, Q_t) \tag{8}$$

where $N(0, \Sigma)$ denotes a normal distribution with mean 0 and covariance matrix Σ.

[7] See Harvey (1990) for a treatment of the Kalman filter.

We first set up the transition equation (8). The conditional mean and variance of X_t are linear functions of X_{t-1}[8],

$$E(X_t|X_{t-1}) = C + DX_{t-1}, \qquad Var(X_t|X_{t-1}) = Q^1 + Q^2 X_{t-1}$$

where the matrices D and Q^2 are diagonal since the processes are independent. We do not observe X_{t-1} and therefore we use the Kalman filter estimate $\hat{X}_{t-1}$ in the calculation of the conditional variance, $Q_t = Q^1 + Q^2 \hat{X}_{t-1}$.

When pricing corporate bonds, it is more convenient to work with the upper-left $K-1 \times K-1$ submatrix of Λ^ν which we denote $\tilde{\Lambda}$. Theoretically, we could treat all the entries in the matrix $\tilde{\Lambda}$ as parameters. However, this adds $(K-1)^2$ parameters to the parameter vector, which is undesirable many when maximizing the likelihood function over the parameter vector. Instead we use a generator matrix empirically estimated using Moody's corporate bond default database for the period 1987-2002 The matrix is shown in Table 1.

$\tilde{\Lambda}$	AAA	AA	A	BBB	BB	B	C
AAA	-0.0976	0.0847	0.0122	0.0007	0	0	0
AA	0.0157	-0.1286	0.1090	0.0028	0.0003	0.0008	0
A	0.0010	0.0267	-0.1012	0.0678	0.0047	0.0010	0
BBB	0.0009	0.0024	0.0669	-0.1426	0.0647	0.0067	0.0009
BB	0	0.0004	0.0066	0.1220	-0.2391	0.1069	0.0031
B	0	0.0004	0.0024	0.0103	0.0672	-0.2037	0.1233
C	0	0	0.0018	0.0070	0.0070	0.0648	-0.0806

Table 1: This Table shows the transition intensity matrix (excluding default state) for corporate bonds estimated using Moody's corporate bond default database for the period 1987-2002. The speculative grade categories are gathered in one state as shown in Table 2 before the matrix is used in the empirical work via the pricing formula (4).

We assume that rating transitions are conditionally Markov and ignore downward drift effects. That is, for given level of $\mu(X_s)$ the intensity of downgrade is a function only of the current state and not of the previous rating history. Results in Lando and Skødeberg (2002) indicate that this may be a reasonable approximation for financial firms.

As mentioned, the data include corporate yields for the rating categories AAA, AA, A, and BBB. All investment grade ratings. The remaining rating

[8]See de Jong (2000).

categories BB, B, and C, which are all speculative grade rating categories, are treated as one rating category denoted SG. The generator matrix in Table 1 is therefore reduced in the following way:

- for the investment grade rating categories the transition intensities for changing rating to BB, B, and C are added and used as the transition for changing rating to SG,
- the intensities for going from BB to investment grade ratings are used as the intensities for going from SG to investment grade. The intensity for a jump to a different rating from SG ($\lambda_{SG,SG}$) is changed such that the last row in the new generator matrix still sums to zero.

$\tilde{\Lambda}$	AAA	AA	A	BBB	SG
AAA	-0.0976	0.0847	0.0122	0.0007	0
AA	0.0157	-0.1286	0.1090	0.0028	0.0011
A	0.0010	0.0267	-0.1012	0.0678	0.0057
BBB	0.0009	0.0024	0.0669	-0.1426	0.0723
SG	0	0.0004	0.0066	0.1220	-0.1291

Table 2: This Table shows the transition intensity matrix intensities from Table 1, where speculative grade states are gathered in one state, SG.

The resulting generator matrix is given in Table 2. The new category SG can be regarded as a "downward adjusted" BB category, because the transitions intensities from BB are kept, while the transition intensities to SG are slightly higher than the original transition intensities to BB. The problems of reducing the generator matrix are concentrated in the SG rating category. If we were to price speculative grade bonds the way of reducing the generator matrix would be problematic, but since we only price AAA, AA, A, and BBB rated bonds the adjusted transition intensities do not cause problems for the modelling. However, care has to be taken when interpreting the SG rating category.

Corporate bonds and swap rates are nonlinear functions of the state variables and we write the observed yields as $y_t = f(X_t)$. A first-order Taylor approximation of $f(X_t)$ around the forecast $\hat{X}_{t|t-h}$,

$$\begin{aligned} f(X_t) &\simeq f(\hat{X}_{t|t-h}) + \hat{B}_t(X_t - \hat{X}_{t|t-h}) \\ &= f(\hat{X}_{t|t-h}) - \hat{B}_t\hat{X}_{t|t-h} + \hat{B}_t X_t, \end{aligned}$$

where

$$\hat{B}_t = \frac{\partial f(x)}{\partial x}|_{x=\hat{X}_{t|t-h}}, \tag{9}$$

yields the matrix B_t in the measurement equation[9].

We assume that all yields are measured with independent errors with identical variance, so $var(\epsilon_t) = \sigma^2 I_{42}$. Furthermore, we assume that the processes are stationary under P(implying $k_i < 0$) and use the unconditional distribution as initial distribution in the Kalman filter recursions. BBB yields are missing for a period but the Kalman filter can easily handle missing observations and we refer to Harvey (1990) p. 143-144. The reason for restricting the Q-mean of all the processes $X_1, ..., X_6$ to be zero is that empirically, not all of the parameters can be estimated. For example in $r = X_1 + X_2$ the mean and α_i of each factor are not separately identified[10]. With this normalization α can be interpreted as the average volatility of each factor. In addition, we added a constant mean to government rate, convenience yield, μ and S processes such that

$$\begin{aligned} r^g(X) &= a + X_1 + X_2, & (10) \\ r(X) &= a + X_1 + X_2 + (e + X_5), & (11) \\ \mu(X) &= b + X_3 + X_4 + c(X_1 + X_2). & (12) \\ S(X) &= d + X_6 & (13) \end{aligned}$$

Restricting D to be a positive CIR process implies the restriction $\alpha_5 = e\beta_5$[11].

The outlined extended Kalman filter does not yield consistent parameter estimates for two reasons. First, in the estimate of $Var(X_t|X_{t-1})$ we use $\hat{X}_{t-1}$ instead of X_{t-1} and set $\hat{X}_{it} = -\frac{\alpha_i}{\beta_i}$ if $\hat{X}_{it} < -\frac{\alpha_i}{\beta_i}$. Nevertheless, Monte Carlo studies in Lund (1997), Duan and Simonato (1999), and de Jong (2000) indicate that the bias of this approximation is negligible. Second, the pricing function f in the measurement equation is linearized around $\hat{X}_{t|t-1}$. In order to asses the possible bias in parameter estimates we conducted a small Monte Carlo experiment suggesting that the approximate Kalman filter works well in estimating our model. Appendix C gives the details of the Monte Carlo experiment along with further estimation details.

[9] It is not necessary to calculate A_t in the linearization since it is not used in the extended Kalman filter.

[10] See de Jong (2000).

[11] The process $Y = e + X$ has dynamics $dY = k(e - Y)dt + \sqrt{\alpha - e\beta + \beta Y} dW$.

5 Empirical Results

We fit 6 different yield curves from three different markets using 6 factors. A natural question is whether the model is able to fit all curves simultaneously. Table 3 shows the mean, standard error, and first-order autocorrelation of the residuals. The average pricing error for all yields is less than half a basis point while the average standard error is less than 8 basis points. The BBB yield curve has the worst fit, which is seen by the largest average standard errors. This suggests that the our specification of the generator matrix enables us to price highly rated corporate bonds well while the pricing of lower rated bonds might be more problematic. However, for our purpose the fit of the corporate bonds is satisfactory. The yield curve with the smallest pricing error is the swap curve and the errors are less or comparable to those of other papers estimating the swap curve. We note that a sign of misspecification of the model is that the first-order correlations of the residuals are strongly positive which is the typical case[12].

[12] See for example Duffie and Singleton (1997) and Collin-Dufresne and Solnik (2001).

	$\epsilon_{0.25}$	$\epsilon_{0.5}$	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	average
Govt										
Mean		1.72	-2.18	1.68	1.38	0.94	-4.87	1.05	3.82	0.442
St. dev.		9.24	8.83	9.48	7.06	5.45	6.58	6.9	8.03	7.7
ρ		0.861	0.926	0.933	0.922	0.91	0.907	0.898	0.869	0.903
AAA										
Mean			-0.9	-2.96	0.8	0.85	-1.84	-2.71	-1.55	-1.187
St. dev.			7.68	6.11	8.69	8.01	7.29	6.95	8.01	7.53
ρ			0.861	0.81	0.912	0.911	0.873	0.892	0.851	0.873
AA										
Mean			2.44	2.34	3.52	2.35	1.29	-0.06	1.78	1.951
St. dev.			9.69	7.75	6.27	7.01	6.45	5.42	6.3	6.98
ρ			0.91	0.876	0.805	0.875	0.828	0.838	0.765	0.843
A										
Mean			0.96	-1.82	0.41	0.08	-2.16	-1.71	2.75	-0.2129
St. dev.			6.81	8.22	4.91	5.08	5.63	5.2	6.37	6.03
ρ			0.819	0.85	0.708	0.787	0.828	0.848	0.779	0.803
BBB										
Mean			-1.29	-0.49	1.28	3.04	0.74	1.27	3.52	1.153
St. dev.			10.04	7.01	9.56	11.8	14.34	14.9	16.37	12
ρ			0.755	0.67	0.632	0.605	0.655	0.707	0.741	0.681
LIBOR										
Mean	1.23									1.23
St. dev.	15.06									15.06
ρ	0.869									0.869
Swap										
Mean				-2.05	0.97	1.16	0.87		-0.6	0.07
St. dev.				8.53	4.53	4.21	4.45		6.12	5.57
ρ				0.933	0.811	0.755	0.705		0.811	0.803

Table 3: This Table shows statistics for the residuals of the government, corporate, LIBOR, and swap rates measured in basis points. The residual is $\epsilon_t = y_t - \hat{y}_t$ where $\hat{y}_t$ the model-implied yield. The means, standard deviations, and first-order autocorrelations ρ are shown.

Parameters of the state variables

	k	θ	α	β	λ	$k*$
X_1	-0.2881	-0.0269	0.0007	0.0051	-10.3813	-0.2351
			(0.000037)	(0.000016)	(5.660690)	(0.000442)
			(0.00008)	(0.00066)	(20.34299)	(0.00476)
X_2	-0.6455	-0.0088	0.0007	0.0005	-8.1534	-0.6418
			(0.000079)	(0.004648)	(7.888439)	(0.022097)
			(0.00012)	(0.00202)	(22.06073)	(0.01372)
X_3	-0.2246	-1.4849	0.4238	0.0000	-0.7868	-0.2246
			(0.075498)	(0.003863)	(0.181224)	(0.004944)
			(0.06711)	(0.00800)	(0.47994)	(0.00852)
X_4	-0.0025	0.0013	0.0001	0.8729	0.0414	-0.0387
			(0.000001)	(0.112667)	(0.006340)	(0.000108)
			(0.03434)	(0.04095)	(0.03584)	(0.00295)
X_5	-0.0066	0.0001	0.0000	0.0011	397.9246	-0.4468
				(0.000031)	(9.771331)	(0.004064)
				(0.00016)	(0.01376)	(0.02232)
X_6	-0.0634	-0.0310	0.0000	0.0001	-355.7386	-0.0234
			(0.000000)	(0.000001)	(28.276065)	(0.008918)
			(0.00000)	(0.00003)	(226.44959)	(0.01891)

Other parameters

	a	b	c	d	e	σ^2
	0.065818	0.359701	-23.855810	0.029284	0.000002	0.000001
	(0.000367)	(0.030184)	(0.008164)	(0.000018)	(0.007488)	(0.000000)
	(0.00050)	(0.04760)	(1.48397)	(0.00034)	(0.02321)	(0.00000)
	ν_1	ν_2	ν_3	ν_4	ν_5	
	0.002178000	0.003490162	0.008656542	0.016696560	0.024827600	
	(0.000173)	(0.000216)	(0.000173)	(0.000620)	(0.001152)	
	(0.00021)	(0.00023)	(0.00044)	(0.00077)	(0.00121)	

Table 4: This Table shows parameter estimates resulting from the Kalman filter estimation described in section 4. The model is

$$r^{govt} = a + X_1 + X_2 \quad (10)$$
$$r^{riskless} = r^{govt} + e + X_5 \quad (11)$$
$$\mu = b + c(X_1 + X_2) + X_3 + X_4 \quad (12)$$
$$\lambda^{LIBOR} = \lambda^{AA} + d + X_6 \quad (13)$$

where the numbers are the corresponding equation numbers in the text. The first set of standard errors are calculated as

$$\hat{\Sigma}_1 = \frac{1}{T}[\hat{A}\hat{B}^{-1}\hat{A}]^{-1}$$

where $\hat{A} = -\frac{1}{T}\sum_{i=1}^{T} \frac{\partial^2 \log l_t(\hat{\theta})}{\partial\theta\partial\theta'}$ and $\hat{B} = \frac{1}{T}\sum_{i=1}^{T} \frac{\partial \log l_t(\hat{\theta})}{\partial\theta}\frac{\partial \log l_t(\hat{\theta})'}{\partial\theta}$ while the second set of standard errors are calculated as

$$\hat{\Sigma}_2 = [T\hat{B}]^{-1}.$$

Note that there are no standard errors on α_5 because of the restriction $\alpha_5 = d\beta_5$ ensuring that $D(X) = e + X_5$ remains positive.

The estimated parameters are given in Table 4. Two sets of standard errors are reported: White (1982) heteroscedasticity-corrected standard errors and standard errors without the correction. The former is theoretically more robust while the latter is numerically more stable and details are given in appendix C. Also, the means of the variables are difficult to estimate reliably and therefore are hard to interpret which is a common problem encountered in for example Duffee (1999) and Duffee and Stanton (2001).

From the table we see that credit risk is inversely related to government rates because the parameter c is significantly negative. This is consistent with research using only Treasury and corporate bond data (Duffee (1999) and Driessen (2005)) but in contrast to research using only swap and Treasury data (Liu, Longstaff, and Mandell (2004)). Using both Treasury, swap and corporate bond data we find a negative relationship as in Collin-Dufresne and Solnik (2001) and we suspect that the result in Liu, Longstaff, and Mandell (2004) is due to the use of the LIBOR - GC repo spread as a proxy for credit risk - a point we will discuss more thoroughly in section 5.1.

Turning to the filtered state variables, the two variables X_1 and X_2 governing the government curve has the usual interpretation as level and slope as seen in Figure 4. More interesting is whether the variables X_3 and X_4 governing the credit risk process μ has a similar interpretation. As we see in Figure 5 X_3 can be interpreted as the mean credit level defined as $\frac{y_7^{AAA}+y_7^{AA}+y_7^{A}}{3} - y_7^g$ and X_4 as the mean credit slope as $\frac{y_7^{AAA}+y_7^{AA}+y_7^{A}}{3} - y_7^g - [\frac{y_1^{AAA}+y_1^{AA}+y_1^{A}}{3} - y_1^g]$ even though the picture is not as convincing as for the government factors. This is to be expected since the a) the credit risk factor also depends on the government rate, and b) the spread between corporate bonds and government rates consists af both a liquidity and credit risk factor and μ accounts only for the credit risk factor.

[Figure 4 about here.]

[Figure 5 about here.]

5.1 Credit risk

The market's perception on credit risk have large effects on swap spreads as documented in Duffie and Singleton (1997) and other papers, although swap rates carry less default risk than AA corporate rates as showed in Collin-Dufresne and Solnik (2001). However, papers separating out the credit risk component in swap spreads has to our knowledge all relied on using proxies for credit risk.

The usual candidate for a credit risk proxy is the spread between LIBOR and GC Repo - which we label the LGC spread. LIBOR rates are rates on unsecured loans between counterparties rated AA on average while GC Repo rates are rates on secured loans and therefore the difference is thought to be due to a credit risk premium. In figure 6 we compare the 3-month LGC spread with the estimated 3-month AA credit risk premium. The 3-month AA credit risk premium on date t is calculated as the difference in basis points between the yield on a 3-month AA corporate bond and a 3-month riskless bond (with no liquidity), while the 3-month LIBOR and GC repo rates are from Bloomberg. The average estimated premium is 10.0 basis points while the average observed LGC spread is 14.7 basis points. The difference in averages is to a large extent due to large difference before the year-end in 1999 and 2000 and if we exclude the last three months before 1999 and 2000 the averages are 10.0 and 12.2. Even though the averages are similar the LGC spread is very volatile while the estimated AA default premium is much more persistent. A possible explanation for the different behavior of the two time series is given in Duffie and Singleton (1997). In their model the LIBOR rate is poorly fitted and they suggest that there might be noncredit factors determining LIBOR rates. Support for this view is given in Griffiths and Winters (2004) who examine one-month LIBOR and find a turn-of-the-year effect. The rate increases dramatically at the beginning of December, remains high during December, and decreases back to normal at the turn-of-the-year, with the decline in rates beginning a few days before year-end. This effect is a liquidity effect unrelated to credit risk. If the GC repo rate does not have a turn-off-the-year effect, the LGC spread will mirror this liquidity effect.

[Figure 6 about here.]

We see the largest difference between the LGC spread and estimated AA credit premium in the last three months before the Millenium Date Change

(Y2K). Three months before Y2K the LGC spread jumps from 11 to 79 basis points. If the jump was caused by general credit risk concerns all LGC spreads for various maturities would jump simultaneously. If the jump was caused by credit risk concerns right after Y2K we would see LGC spreads remaining high until Y2K. Neither is happening as we see in Figure 7. As argued in Sundaresan and Wang (2004), due to concern around Y2K, lenders in the interbank market wanted a premium to lend cash due shortly after Y2K and the jump is therefore due to a liquidity premium on short-term lending. Because the 3-month credit risk premium in our model is estimated on basis of a range of yields and maturities, we see in Figure 7 that the premium is practically unaffected by the Y2K.

[Figure 7 about here.]

Consequently, we view the LGC spread as an inappropriate proxy for credit risk. Liu, Longstaff, and Mandell (2004) proxy credit risk with this spread and assume that the spread is observed without error. This implies that the credit risk component inherents the properties of the LGC spread in being volatile and rapidly mean-reverting. This in turn implies that long-term swap spreads are only weakly affected by fluctuations in the credit risk component. In our model credit spread fluctuations are not as mean-reverting and therefore have a larger impact on long-term swap spreads.

5.2 The Swap factor

In contrast to earlier literature on swap spreads we allow for a unique liquidity factor for the swap market. This factor has a strong impact on swap spreads both cross-sectionally and in the time series dimension.

We can calculate the effect in basis points of the swap factor cross-sectionally by the formula $-\frac{1}{T}\log(E_t(\exp(-\int_t^{t+T} S(u)du))$ and transform the effect into basis points in par rates[13]. The result is shown in Table 5. We see that there is a significant maturity premium in interest rate swap relative to government bonds. While this premium on average is approximately zero at a maturity of two years the average premium is 15.4 basis points at seven

[13]This is actually an approximation. We can find the exact effect of the swap factor by calculating swap rates with and without the swap factor and find the effect as the difference between the two calculated swap rates. However, the approximation differs less than one basis point from the exact calculation so for simplicity we use the approximation.

years[14]. The difference between the 7-year premium and the 2-year premium fluctuates between 15.2 and 18.0 basis points so the maturity premium is not very time-varying. This evidence supports the existence of a risk premium for holding swaps relative to government bonds - a point also discussed by He (2001).

maturity	2	3	4	5	7
average effect	-0.8	2.6	5.9	9.1	15.4

Table 5: This table shows the average effect in basis points of the swap factor on swap rates across maturities. The effect of the swap factor on maturity T at time t is calculated as $-\frac{1}{T}\log(E_t(\exp(-\int_t^{t+T} S(u)du))$.

While the factor has a relatively stable impact on different maturity relative to each other, we observe in Figure 8 that the factor varies greatly during the estimation period with a difference of around 50 basis points from the high in the middle of 2000 to the low in the end of 2002. We see that the factor fluctuates more after the turn of the Millenium than before.

[Figure 8 about here.]

A reason for the fluctuations of the swap factor is found in the US MBS market. The US mortgage market has more than doubled in size since 1995. Also, relative to the Treasury market the MBS market has grown and in 2000 the size surpassed the Treasury market as noted in BIS (2003). Due to the prepayment risk embedded in MBSs - borrowers are allowed to prepay the mortgage which creates uncertainty regarding the timing of cash flows of MBSs - movement in interest rates often result in significant changes in the option-adjusted duration of an MBS. When interest rates drop, borrowers can refinance their mortgages by exercising their option to call the mortages at par value.

The two largest mortgage holders, Fannie May and Freddie Mac, held around a third of the total MBS market by the end of 2002 according to Perli and Sack (2003). Both institutions have guidelines available to the public regarding their hedging of duration. In order to hedge interest rate risk they seek to keep the net total duration of their balance sheet within a

[14] In an earlier version of the paper we estimated the model without the swap factor and the swap rates were systematically overfitted at short maturities and underfitted at long maturities.

range around zero by hedging their holdings with Treasury securities, interest rate swaps or related derivatives such as swaptions and caps. Other market participants follow a similar trading strategi although to a lesser extent. As an example, if interest rates drop, duration on MBSs drop and Fannie Mae/Freddie Mac offset the duration drop on their holdings by entering as the fixed-receiver in an interest rate swap.

The research of the impact of MBS hedging on interest rates has been modest. Perli and Sack (2003) examine the effect of MBS hedging in the volatility of the ten-year swap rate and find that the hedging amplifies the volatility considerably. Duarte (2005) find that MBS refinancing helps explaining swaption prices considerable, especially during periods of high refinancing activity. From the end of 2000 and until the end of his sample in September 2003 was a period of high activity and incorporating MBS information reduces swaption pricing errors considerably.

In modeling several markets simultaneously we can ask the question whether MBS hedging affects swap rates and Treasury rates differently. If the two markets are affected by MBS hedging to the same extent, the swap spread should be unchanged. However, as noted in BIS (2003) the concentration of OTC hedging activity in a small number of dealers in the swap market seems to make the market more vulnerable to a loss of liquidity. If a few dealers breach their risk limits and cut back on the market-making activity the whole market loses liquidity. Therefore falling (rising) option-adjusted duration can cause swap rates to fall below (rise above) their long run level. Hence, if the swap market is more vulnerable to MBS hedging, we see an isolated effect of option-adjusted duration on swap spreads when controlling for credit risk and convenience yield in Treasury securities.

In Figure 9 we see the demand factor along with a linear function of Lehman modified duration index for mortgage backed securities obtained from Datastream. We see that before Y2000 there is no relation (correlation -0.01), while after there is a strong relation (correlation 0.86) amounting to as much as 40 basis points. In the pre-Y2K period the year of 1998 was a period with a considerable refinancing wave but we see that it did not strongly affect the liquidity of the swap market relative to the Treasury market. In contrast the large refinancing wave beginning in the end of 2000 and lasting until the end of the sample resulted in swap rates falling more than Treasury rates. In November 2002 swap rates deviated as much as 32 basis points from their average. The fact that the swap market reacted differently in the two refinancing waves before and after Y2K is possibly due to nature of hedging during the two waves. As Wooldridge (2001) notes, non-government securities were routinely hedged with government securities until the 1998 crisis. However, periodic breakdowns in the normally stable relationship between government and non-government securities lead many market participants to switch hedging instruments from government to non-government securities such as interest rate swaps. Reinhart and Sack (2002) analyze the properties of the 10-year swap spread and estimate an idiosyncratic swap factor similar to our estimated swap factor. In their data set, which runs until the second half of 2001, they find a larger influence of the swap factor in 2001 and conjecture that it is due to the use of swaps as hedging instruments in the MBS market. We confirm this conjecture using a swap factor that is part of a full pricing model. This enables us to capture both the time series behavior and the term structure effects of the swap factor.

[Figure 9 about here.]

5.3 The Treasury Factor

Finally, we turn to the estimated Treasury premium. The premium is a convenience yield on holding Treasury securities arising from among other things a) repo specialness due to the ability to borrow money at less than the General Collateral rates(Duffie (1996)), b) that Treasuries are a desired mechanisn for hedging interest rate risk, c) that Treasury securities must be purchased by financial institutions to fulfill regulatory requirements, d) that the amount of capital required to be held by a bank is significantly smaller to support an investment in Treasury securities relative to other near default-free securities, and to a lesser extent e) a liquidity premium interpreted as the ability to absorb a larger number of transactions without dramatically affecting the price. In our preliminary fitting of the model we could not obtain a joint fit of corporate, Treasury and swap curves without the Treasury factor. Without this factor the corporate bonds and swaps are systematically mispriced. In our model the process $L = e + X_5$ is a measure in basis points of the Treasury premium at the very short end of the yield curve. For longer-term maturities we can estimate the effect in basis points of the Treasure factor since the price of a riskless bond is given as

$$P(t,T) = E_t(\exp(-\int_t^T r^g(s) + L(s)ds)) = P^g(t,T)E_t(\exp(-\int_t^T L(s)ds)),$$

and therefore

$$y(t,T) = y^g(t,T) - \frac{1}{T-t}\log(E_t(\exp(-\int_t^T L(s)ds)). \tag{14}$$

Because we have endogenously estimated the Treasury premium without any proxies for the riskless rate, we are able to compare it with existing proxies for the premium in the literature and assess the quality of these proxies.

We can compare the impact of the estimated Treasury factor in the short end with the proxy used in Liu, Longstaff, and Mandell (2004), the spread between GC repo and government. GC repo rates are rates for collateralized loans in the repo market where a broad range of securities can be used as collateral. Because the loans are collateralized they are essentially risk free and therefore proxy the risk free rate as suggested by Longstaff (2000). Repo rates are only available at short maturities, typically three months or less, so we use the spread between 3-month GC repo collected from Bloomberg and the secondary market rate of the 3-month Treasury bill collected from the Federal Reserve[15]. The estimated 3-month Treasury premium and the 3-month GC repo- bill spreads are graphed in Figure 10. On average the estimated 3-month Treasury premium is 48,3 basis points while the repo-bill spread is 43,8 basis points, so the estimated Treasury premium is slightly larger than the repo - bill spread on average. The figure shows that the two spreads largely agree on the general level of the factor but there are periods where the difference is quite large and the Treasury premium is less volatile than the repo-bill spread. A plausible explanation for the lower volatility of the Treasury premium could be that on each date the short-end effect of the factor is implied out from several yield curves on a whole range of maturities and possible noise contaminating the repo-bill spread is eliminated. The noise in the repo - bill spread could be idiosyncratic effects in short-term interest rates. Duffee (1996) finds that there is a common movement in short-term Treasury bill yields that is unrelated to movements in longer-maturity Treasury notes and bonds, and therefore the use of the short-term repo-government spread as a proxy for the Treasury premium results in an inaccurate measure. This could explain that the Treasury component in Liu, Longstaff, and Mandell (2004) is negative for a large part of their sample - a result that is hard to interpret.

[15]This differs slightly from Liu, Longstaff, and Mandell (2004) who use 3-month GC repo and government bonds of maturity 2 years and longer to estimate the Treasury factor.

[Figure 10 about here.]

We are able to estimate the effect of the Treasury factor at various maturities by using (14) and can compare the resulting term structure of Treasury premia with proxies suggested in the literature.

He (2001) proposes to estimate yield curves using on-the-run respectively off-the-run Treasury bonds and proxy a term structure of the Treasury factor by differencing the off-the-run and on-the-run curves. However, as Krishnamurthy (2002) points out the yield spread between an on-the-run and off-the-run bond displays cyclical behavior between auction dates[16]. In addition, off-the-run bonds trade at a higher price than off-off-the-run bonds, off-off-the-run bonds trade at a higher price than off-off-off-the-run bonds, and so forth, so even off-the-run bonds contain a sizeable Treasury component. Therefore, the spread between off-the-run and on-the-run bonds is only a fraction of the total component in Treasury bonds.

An alternative measure of the Treasury component is the spread between Refcorp bonds/ strips and the government curve suggested in Longstaff (2004) and Longstaff et al. (2005). Refcorp bonds are implicitly backed by the U.S. Treasury and therefore the credit risk premium is essentially zero. Six Refcorp bonds are issued maturing in 2019, 2020, 2021, and 2030 and the principal amounts outstanding range from 4.5 to 5.5 billion dollars[17]. Reinhart and Sack (2002) proxy the Treasury factor in the 10 year Treasury rate by the difference between a Refcorp bond maturing 2020 and a comparable government rate[18]. However, since Refcorp bonds with less than 15 years to maturity do not exist we cannot use Refcorp bonds directly as a measure of the Treasury factor in the short and medium end of the yield curve. As with Treasury bonds, Refcorp bonds can be held in stripped form so a whole range of Refcorp zero coupon bonds exists. From these a yield curve is estimated daily and readily available at Bloomberg. In Figure 11 the Refcorp-government spread is graphed for maturities 1 year and 7 year. We see very large differences between the 1 and 7 year proxies. The 1 year Treasury factor proxy is around 100 basis points in the first half of 2001 while

[16]Generally, the spread is highest after an auction date and decreases until the next auction date.

[17]For more information see Longstaff (2004).

[18]Since there are no Refcorp bonds with significantly shorter time to maturity it is necessary to proxy the Treasury factor in the 10 year Treasury rate by a spread at the 20-25 year segment.

the 7 year proxy averages about 20 basis points in same period. For a period in 2002 the 1 year spread is even negative at the level of about -20 basis points. This suggests that the spread between the Refcorp and government yield curves is hard to interpret as a term structure of the Treasury factor.

[Figure 11 about here.]

If we assume that Refcorp strips are unbiased but highly correlated measures of riskless bonds without the Treasury factor at time t,

$$y^{Refcorp}(t,T) = y^{riskless}(t,T) + \epsilon_t, \qquad E(\epsilon_t) = 0, cor(\epsilon_t, \epsilon_{t-1}) = \rho$$

we can average the Refcorp - government spread over time and get an approximation of the average Treasury factor on the maturity $T - t$,

$$E(y^{Refcorp}(t,T)) = E(y^{riskless}(t,T)).$$

Figure 12 shows the average Refcorp - government curve along with the average estimated Treasury factor curve. Both yield curves are downward sloping and on average the estimated term structure of Treasury factor is downward-sloping with 41.2 basis points at a maturity of 1 year and 15.6 basis points at 7 years. This is consistent with Driessen (2005) who also finds a downward-sloping term structure of liquidity using a different methodology.

[Figure 12 about here.]

If we look at the correlation between the Refcorp - government spread and our estimated Treasury factor across maturities, we see in Table 6 that the Refcorp - government spread does seem to be a poor measure of the Treasury factor in the short end of the yield curve while it is more precise - albeit still noisy - at a maturity of 5 years.

maturity	0.5	1	2	3	4	5	7
correlation	-0.03	0.37	0.41	0.29	0.44	0.64	0.32

Table 6: This table shows the correlation between the Refcorp - government spread and estimated liquidity at various maturities. Estimated Treasury factor at maturity T is calculated according to formula (14) as $-\frac{1}{T-t}\log(E_t(\exp(-\int_t^T L(s)ds))$ while the Refcorp and government yields are continuously compounded yields from Bloomberg.

5.4 Decomposing swap spreads

We have analyzed three components in swap spreads - a Treasury, a credit risk, and a swap component - separately and now turn to the joint effect of the components on swap spreads.

In Figure 13 the estimated 5-year swap spread is decomposed into the three components. We see that while the Treasury factor accounted for the largest part of the swap spread in first half of the estimation period, the size of the factor diminished in the second half of the period. This suggests that the importance of the Treasury factor has lessened during the estimation period. The swap factor accounted for a relatively small part of the swap spread before year 2000, while the factor has contributed to a contraction of the swap spread in 2000-2003. From August 2002 and until the end of the estimation period in February 2003 the swap component was negative by as much as 20 basis points suggesting a strong pressure to enter as the fixed-receiver in an interest rate swap. Indeed, IMF (2003, p. 17) reports that "In August 2002, the duration gap between Fannie Mae's assets and liabilities widened to minus 14 months, as falling interest rates increased likely prepayment rates and thus shortened the expected duration of its mortgages. This gap prompted Office of Federal Housing Enterprise Oversight to require an action plan to correct this imbalance and to monitor Fannie Mae's maintenance of its duration gap for the following six months before it declared itself satisfied in April 2003". As noted in the previous section, Fannie Mae could eliminate the duration gap by entering as the fixed-receiver pressuring down swap rates. While the credit risk factor had a relatively small impact on the 5-year swap spread prior to Year 2000 we see that the factor has grown in importance after Year 2000. The larger impact of credit risk is due to steeper credit spread slopes in the corporate bond market as seen on the right graph in Figure 5. This highlights the importance of incorporating corporate bonds in modeling swap spreads.

[Figure 13 about here.]

In Figure 14 we see the average effect of the three components on swap spreads across maturity. The Treasury component accounts for the largest part of the swap spread in the short end of the swap curve while the size of the Treasury factor decreases with maturity. As noted in the previous section the size of the swap factor points to a maturity premium on swap rates. Since the average premium at a maturity of seven years is 15.4 basis points while it it is -0.8 basis points at maturity two years, the average premium per maturity year is estimated to be 3.2 basis points. Finally, the average effect of the credit risk component is 28.9 basis points at maturity 7 years and 17.2 basis points at maturity 2 years. Thus, the size of the average credit risk factor is increasing with maturity.

[Figure 14 about here.]

6 Conclusion

We have proposed a joint pricing model for Treasury securities, corporate bonds and swap rates using six latent factors, and we show that the model is capable of fitting the term structures of Treasury bonds, an index of investment grade corporate bonds and swap rates. This allows us to decompose swap spreads into three components: A credit risk component, a swap factor, and a Treasury factor. Because we include information from the corporate bond market, the decomposition can be performed without reference to proxies. We are therefore able to 1) assess the accuracy of some commonly used proxies and 2) decompose swap spreads in two dimensions - across time and maturity. As far as the accuracy of proxies is concerned, we find the following. First, the credit risk factor is important for understanding swap spreads, but it is better captured by the information in the corporate bond market rather than the commonly used proxy LIBOR - GC repo spread. Second, we find that the Treasury factor - often interpreted as a convenience yield to owning Treasuries - is less volatile than the proxies GC repo - Treasury Bill and Refcorp - government. Our decomposition of swap spreads across time and maturities shows that the credit risk component is important and it is on average increasing with the maturity of the swap. In addition, the swap spread is influenced by an idiosyncratic swap factor. Cross-sectionally, the average effect of the factor is 15 basis points at seven years and zero basis points at two years. In the time series dimension, the swap factor has a strong correlation with hedging activity in the MBS market after 2000. Finally, we find that the Treasury factor accounts for most af the swap spread in the short end but is decreasing with maturity.

A A Result on Univariate Affine Processes

For the univariate affine process

$$dX_t = k(X_t - \theta)dt + \sqrt{\alpha + \beta X_t} dW, \tag{15}$$

we know from Duffie, Pan, and Singleton (2000) that there exists A and B such that

$$E_t(e^{-\int_t^T c_1 X_u du} e^{c_2 X_T}) = e^{A(t,T)+B(t,T)X_t}. \tag{16}$$

Christensen (2002) has derived explicit expressions for A and B and they are given as

$$\begin{aligned}
B(t,T) &= \frac{-2c_1(e^{\gamma(T-t)} - 1) + c_2 e^{\gamma(T-t)}(\gamma + k) + c_2(\gamma - k)}{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)} - 1)} \\
A(t,T) &= \frac{-2k\theta}{\beta} \ln\left(\frac{2\gamma e^{\frac{1}{2}(\gamma-k)(T-t)}}{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)} - 1)}\right) \\
&\quad + \frac{1}{2}\alpha \frac{(2c_1 + c_2(\gamma - k))^2}{(\gamma + k + c_2\beta)^2}(T - t) \\
&\quad + \frac{2\alpha k}{\beta^2} \ln\left(\frac{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)} - 1)}{2\gamma}\right) \\
&\quad - \frac{2\alpha}{\beta} \frac{(e^{\gamma(T-t)} - 1)(c_1 - kc_2 - \frac{1}{2}\beta c_2^2)}{2\gamma + (\gamma - k - c_2\beta)(e^{\gamma(T-t)} - 1)} \\
\gamma &= \sqrt{k^2 + 2\beta c_1} \qquad \text{for } c_1 > -\frac{k^2}{2\beta}.
\end{aligned}$$

B Pricing Formulas

B.1 Riskless Bonds

The riskless rate is given in equation (11) as

$$r = X_1 + X_2 + X_5.$$

As noted in section 4, in order to have identification of the parameters we set the means of the processes to zero and add a constant,

$$r = (a + X_1 + X_2) + (e + X_5).$$

Prices of riskless bonds are

$$
\begin{aligned}
P(t,T) &= E_t(\exp(-\int_t^T r(u)du)) = \\
&= e^{-(a+e)(T-t)} e^{-\int_t^T X_{1u}du} e^{-\int_t^T X_{2u}du} e^{-\int_t^T X_{5u}du} \\
&= e^{-(a+e)(T-t)} e^{A^1(T-t)+B^1(T-t)X_{1t}} e^{A^2(T-t)+B^2(T-t)X_{2t}} e^{A^5(T-t)+B^5(T-t)X_{1t}} \\
&= e^{A(T-t)+\mathbf{B(T-t)}'X_t},
\end{aligned}
$$

where we have used the independence of the processes and a special case of the result in appendix A, and A and $\mathbf{B}$ are given as

$$
\begin{aligned}
A(T-t) &= -(a+e)(T-t) + A^1(T-t) + A^2(T-t) + A^5(T-t), \\
\mathbf{B(T-t)}' &= (B^1(T-t), B^2(T-t), 0, 0, B^5(T-t), 0).
\end{aligned}
$$

B.2 Government Bonds

In equation (10) the government rate is given as

$$r^g = X_1 + X_2.$$

For identification this becomes

$$r^g = a + X_1 + X_2,$$

and the prices of government bonds are

$$P^g(t,T) = e^{A^g(T-t)+\mathbf{B^g(T-t)}'X_t},$$

where $A(T-t)^g = -a(T-t) + A^1(T-t) + A^2(T-t)$ and $B^g(T-t)' = (B^1(T-t), B^2(T-t), 0, 0, 0, 0)$ are derived exactly as in the riskless bond case.

B.3 Corporate Bonds

In the pricing of corporate bonds we choose to work with a generator matrix excluding default states,

$$
\tilde{A}_X(s) = \tilde{\Lambda}^\nu \mu(X_s) = \begin{pmatrix} -\lambda_1 - \nu_1 & \lambda_{12} & \cdots & \lambda_{1,K-1} \\ \lambda_{21} & -\lambda_2 - \nu_2 & \cdots & \lambda_{2,K-1} \\ \vdots & & \ddots & \vdots \\ \lambda_{K-1,1} & \cdots & \cdots & -\lambda_{K-1,K-1} - \nu_{K-1} \end{pmatrix} \mu(X_s),
$$

for notational reasons. We can decompose $\tilde{\Lambda}^\nu$ into $\tilde{\Lambda}^\nu = \tilde{B}\tilde{D}\tilde{B}^{-1}$, where $\tilde{D}$ is a diagonal matrix with the eigenvalues of $\tilde{\Lambda}^\nu$ in the diagonal and $\tilde{B}$ is a $K-1 \times K-1$ matrix with columns given by the $K-1$ eigenvectors of $\tilde{\Lambda}^\nu$. The price of a corporate bond with rating i can be written as

$$v^i(t,T) = \sum_{j=1}^{K-1} -[B]_{ij}[B^{-1}]_{j,K} E_t(e^{\int_t^T \tilde{D}_{jj}\mu(X_u)-r(X_u)du}) \tag{17}$$

according to Lando (1998), so we have $c_{ij} = -[B]_{ij}[B^{-1}]_{j,K}$ and $d_j = \tilde{D}_{jj}$ in (4). From the specification in (11) and (12) we have that $r = a + X_1 + X_2 + (e + X_5)$ and $\mu = b + X_3 + X_4 + c(X_1 + X_2)$ so we can solve the conditional expectation

$$\begin{aligned}
&E_t(e^{\int_t^T \tilde{D}_{jj}\mu(X_u)-r(X_u)du}) = \\
&e^{-(T-t)(a+e-\tilde{D}_{jj}b)} E_t(e^{-\int_t^T((1-c\tilde{D}_{jj})X_{1u}+(1-c\tilde{D}_{jj})X_{2u}+(-\tilde{D}_{jj})X_{3u}+(-\tilde{D}_{jj})X_{4u}+X_{5u}du)}) = \\
&e^{-(T-t)(a+e-\tilde{D}_{jj}b)} \prod_{i=1}^{2}[E_t(e^{-\int_t^T(1-c\tilde{D}_{jj})X_{iu}du})] \times \\
&\qquad\qquad \prod_{i=3}^{4}[E_t(e^{-\int_t^T(-\tilde{D}_{jj})X_{iu}du})]E_t(e^{-\int_t^T(X_{5u}du)}) = \\
&e^{-(T-t)(a+e-\tilde{D}_{jj}b)} \prod_{i=1}^{5}[e^{A_i^j(T-t)+B_i^j(T-t)X_{it}}] = e^{A^j(T-t)+B^j(T-t)'X_t},
\end{aligned}$$

where

$$\begin{aligned}
A^j(T-t) &= -(T-t)(a+e-\tilde{D}_{jj}b) + \sum_{i=1}^{5}[A_i^j(T-t)], \\
B^j(T-t)' &= (B_1^j(T-t), ..., B_5^j(T-t), 0).
\end{aligned}$$

B.4 Swap Rates

To price interest rate swaps we value the floating rate payments and fixed rate payments separately. In the following we assume that the floating rate is paid quarterly while the fixed rate is paid semi-annually. With n being the maturity of the swap in quarters of a year the present value of the floating rate payments in the swap is

$$E_t^Q\Big[\sum_{i=1}^{n} \quad e^{-\int_t^{t_i} r_u du}\big(\tfrac{1}{v^{LIB}(t_{i-1},t_i)} - 1\big)\Big], \tag{18}$$

while the present value for the fixed rate payments are

$$\frac{F(t,T)}{2}\sum_{i=1}^{\frac{n}{2}} P(t,t_{2i}). \tag{19}$$

In evaluating the present value of the floating rate payments we follow the idea outlined in Duffie and Liu (2001). The i'th floating rate payment can be rewritten as

$$E_t^Q\big[e^{-\int_t^{t_i} r_u du}\big(\frac{1}{v^{LIB}(t_{i-1},t_i)} - 1\big)\big] = E_t^Q\big[e^{-\int_t^{t_i} r_u du}\big(\frac{1}{v^{LIB}(t_{i-1},t_i)}\big)\big] - P(t,t_i). \tag{20}$$

Assumption (5) and (6) gives

$$\begin{aligned} v^{LIB}(t_{i-1},t_i) &= E_{t_{i-1}}^Q\big(e^{-\int_{t_{i-1}}^{t_i} \nu_2\mu(X_u)+r_u+d+X_6 du}\big) \qquad (21)\\ &= e^{-0.25(a+d+e+\nu_2 b)} \prod_{j=\{1,2,5,6\}} E_{t_{i-1}}^Q\big(e^{-\int_{t_{i-1}}^{t_i} X_{ju} du}\big) \prod_{j=\{3,4\}} E_{t_{i-1}}^Q\big(e^{-\int_{t_{i-1}}^{t_i} \nu_2 c X_{ju} du}\big)\\ &= e^{-0.25(a+d+e+\nu_2 b)} \prod_{j=1}^{6} e^{\overline{A}_j(0.25)+\overline{B}_j(0.25)X_{jt_{i-1}}} = e^{\overline{A}(0.25)+\overline{\mathbf{B}}(0.25)'X_{t_{i-1}}}, \end{aligned}$$

where

$$\begin{aligned} \overline{A}(0.25) &= -0.25(a+d+e+\nu_2 b) + \sum_{j=1}^{6}\overline{A}_j(0.25),\\ \overline{\mathbf{B}}(0.25)' &= (\overline{B}_1(0.25), ..., \overline{B}_6(0.25)). \end{aligned}$$

Now, using the law of iterated expectations the expectation in the i'th floating rate payment given in equation (20) is

$$\begin{aligned} E_t^Q\big[e^{-\int_t^{t_i} r_u du}\frac{1}{v^{LIB}(t_{i-1},t_i)}\big] &= E_t^Q\big[e^{-\int_t^{t_i} r_u du}e^{-\overline{A}(0.25)-\overline{\mathbf{B}}(0.25)'X_{t_{i-1}}}\big]\\ &= e^{-\overline{A}(0.25)}E_t^Q\big[e^{-\int_t^{t_{i-1}} r_u du}e^{-\overline{\mathbf{B}}(0.25)'X_{t_{i-1}}}E_{t_{i-1}}^Q\big[e^{-\int_{t_{i-1}}^{t_i} r_u du}\big]\big], \end{aligned}$$

and since

$$E^Q_{t_{i-1}}\big[e^{-\int_{t_{i-1}}^{t_i} r_u du}\big] = e^{-0.25(a+e)} \prod_{j=\{1,2,5\}} E^Q_{t_{i-1}}(e^{-\int_{t_{i-1}}^{t_i} X_{ju} du}) = e^{\overline{\overline{A}}(0.25)+\overline{\overline{\mathbf{B}}}(0.25)' X_{t_{i-1}}},$$

where

$$\begin{aligned}
\overline{\overline{A}}(0.25) &= -0.25(a+e) + \sum_{j=\{1,2,5\}} \overline{A}_j(0.25), \\
\overline{\overline{\mathbf{B}}}(0.25) &= (\overline{B}_1(0.25), \overline{B}_2(0.25), 0, 0, \overline{B}_5(0.25), 0),
\end{aligned}$$

we have

$$\begin{aligned}
& E^Q_t\big[e^{-\int_t^{t_i} r_u du} \frac{1}{v^{LIB}(t_{i-1}, t_i)}\big] = e^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)} E^Q_t\big[e^{-\int_t^{t_{i-1}} r_u du} e^{(\overline{\overline{\mathbf{B}}}(0.25)-\overline{\mathbf{B}}(0.25))' X_{t_{i-1}}}\big] \\
&= e^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)-(t_{i-1}-t)(a+e)} * \\
&\qquad E^Q_t\big[e^{-\int_t^{t_{i-1}} X_{1u}+X_{2u}+X_{5u} du} e^{-\overline{B}_3(0.25)X_{3t_{i-1}}-\overline{B}_4(0.25)X_{4t_{i-1}}-\overline{B}_6(0.25)X_{6t_{i-1}}}\big] \\
&= e^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)-(t_{i-1}-t)(a+e)} \prod_{j=\{1,2,5\}} E^Q_t\big[e^{-\int_t^{t_{i-1}} X_{ju} du}\big] \prod_{j=\{3,4,6\}} E^Q_t\big[e^{-\overline{B}_j(0.25)X_{jt_{i-1}}}\big] \\
&= e^{\overline{\overline{A}}(0.25)-\overline{A}(0.25)-(t_{i-1}-t)(a+e)} \prod_{j=1}^{6} e^{\overline{\overline{\overline{A}}}_j(t_{i-1}-t)+\overline{\overline{\overline{B}}}_j(t_{i-1}-t)X_{jt}} \\
&= e^{A^s(t_{i-1}-t)+\mathbf{B^s}(t_{i-1}-t)' X_t},
\end{aligned}$$

where

$$\begin{aligned}
A^s(t_{i-1}-t) &= 0.25\nu_2 b - \sum_{j=3}^{4} \overline{A}_j(0.25) - (t_{i-1}-t)(a+e) + \sum_{j=1}^{6} \overline{\overline{\overline{A}}}_j(t_{i-1}-t), \\
\mathbf{B^s}(t_{i-1}-t)' &= (\overline{\overline{\overline{B}}}_1(t_{i-1}-t), ..., \overline{\overline{\overline{B}}}_6(t_{i-1}-t)).
\end{aligned}$$

Inserting this in formula (18) for the floating rate payments,

$$\sum_{i=1}^{n} (e^{A^s(t_{i-1}-t)+\mathbf{B^s}(t_{i-1}-t)' X_t} - P(t, t_i)),$$

and equating the present value of the fixed and floating rate payments we get the swap rate

$$F(t,T) = \frac{2\sum_{i=1}^{n}(e^{A^s(t_{i-1}-t)+\mathbf{B^s}(t_{i-1}-t)' X_t} - P(t,t_i))}{\sum_{i=1}^{\frac{n}{2}} P(t, t_{2i})}.$$

When calculating the current three-months LIBOR rate we allow for downgrades, so it is calculated as

$$L(t, t+0.25) = \frac{1}{v^{LIB}(t, t+0.25)} - 1$$

where

$$\begin{aligned} v^{LIB}(t, t+0.25) &= \sum_{j=1}^{K-1} -[B]_{ij}[B^{-1}]_{j,K} E_t(e^{\int_t^T \tilde{D}_{jj}\mu(X_u)-(r(X_u)+d+X_{6u})du}) \\ &= \Big(\sum_{j=1}^{K-1} -[B]_{ij}[B^{-1}]_{j,K} E_t(e^{\int_t^T \tilde{D}_{jj}\mu(X_u)-r(X_u)du}) \Big) E_t(e^{-\int_t^T d+X_{6u}du}) \\ &= v^{AA}(t, t+0.25) E_t(e^{-\int_t^T d+X_{6u}du}). \end{aligned}$$

C Estimation Details

C.1 Monte Carlo Experiment

In the main text we noted that a proof of consistent parameter estimates cannot be proved partly due to the linearization in the measurement equation in the Kalman filter. In this appendix we set up a small Monte Carlo experiment in order to assess the possible bias due to the linearization. The experiment is set up as follows. We assume that the government short interest rate r is constant and that liquidity is assumed to be identical zero while the factor $\mu(X)$ is assumed to be one-dimensional. Since we focus on the possible bias arising from the linearization in the measurement equation we assume that X is Gaussian, i.e.

$$dX_t = k^*(X_t - \theta^*)dt + \sigma dW_t^Q.$$

Since the risk premium λ does not enter the pricing functions we will regard it as fixed in the simulations. Besides the three parameters k^*, θ^*, and σ there is the magnitude of the measurement error σ_ϵ and the five default intensity parameters $\nu_1, ..., \nu_5$ to estimate. In each simulation run the latent Gaussian variable X is simulated using that $X_t|X_{t-1}$ is normal distributed. At time t the observed yields are calculated using the pricing formulas for swap rates and corporate yields and adding independent normal distributed errors with variance σ_ϵ^2 to each yield. Finally, the parameters are estimated maximizing the likelihood function in the Kalman filter with the pricing function linearized around $\hat{X}_{t|t-1}$. The process X is also estimated in the linearized Kalman filter using the estimated parameters. This procedure is repeated 500 times. The parameters are chosen at reasonable values and in Table 7 we see the result of the parameter estimation. It is seen that there does not appear to be any significant bias in the parameter estimates and the standard errors on the parameter estimates are quite small, so for parameter estimation the extended Kalman filter can be considered to work very well. The filtered estimate of X at time t, $\hat{X}_t$ is averaged across the 500 simulations for each $t = 0, .., 323$ and graphed in figure 15. The average true X process is also depicted in the Figure. The standard error of the average estimated X at time t is calculated as the standard error of the difference between the true and estimated process across the simulations. We see from the Figure that the average filtered estimate $\hat{X}$ match the average true process X quite well. The estimate $\hat{X}$ might overestimate X slightly with the average difference across simulations and time being 0.0022 but for all practical purposes the bias is negligible[19]. The evidence of the Monte Carlo experiment suggests that the approximate Kalman filter works well in estimating our model.

[19]If we take the median of $\hat{X}_t - X_t$ at time t instead of the mean, the average difference across time is 0.0017.

Parameter	True value	Mean (Std.Err)
k^*	-0.1000	-0.1006 (0.0062)
θ^*	1.000	1.002 (0.028)
σ	0.3000	0.3003 (0.0153)
σ_ϵ	0.000900	0.0008995 (0.0000068)
ν_1	0.002000	0.001998 (0.000097)
ν_2	0.005000	0.004994 (0.000195)
ν_3	0.010000	0.009985 (0.000407)
ν_4	0.02000	0.01997 (0.00083)
ν_5	0.03000	0.02995 (0.00112)

Table 7: Results of a Monte Carlo experiment for the QML estimator of a one-factor Vasicek model $d\mu_t = k^*(\mu_t - \theta^*)dt + \sigma dW_t^Q$. $\nu_i \mu_t$ is the recovery-adjusted hazard rate for rating class i. The number of simulation runs is 500. In each simulation, 322 weekly observations on 2-, 3-, 4-, 5-, 7-year swap rates, 1-, 2-, 3-, 4-, 5-, 6-, 7-year AAA-, AA-, A-, BBB-yields and 3-month AA yield are observed. Measurement error $\sigma_\epsilon I_{34}$ is added to the observations.

[Figure 15 about here.]

C.2 Optimization

The optimization of the likelihood function is complicated due to two reasons. First, in the Kalman filter recursions it might happen that $\hat{X}_t < -\frac{\alpha}{\beta}$ in which case we set $\hat{X}_t = -\frac{\alpha}{\beta}$ to ensure that $Var(X_t|X_{t-1})$ remains positive definite. In this case we follow Duffee and Stanton (2001) and set the log-likelihood function to a large negative number, so it will not be optimal hit the boundary. Second, the likelihood function has a large number of local maxima as is common in these types of models. We employ the usual procedure of repeatedly generating a random vector of starting values and maximize the log-likelihood function. This was done 100 times using the Nelder-Mead maximization algorithm and the largest of the 100 resulting values was chosen. For the 10 largest values we compared the parameter values and they were generally in the same range. More importantly in regard to the conclusions in our paper, the filtered processes were very close to each other across the 10 values.

C.3 Standard Errors

Because the log-likelihood function is misspecified for non-Gaussian models, a robust estimate of the variance-covariance matrix can be found using White (1982) as

$$\hat{\Sigma}_1 = \frac{1}{T}[\hat{A}\hat{B}^{-1}\hat{A}]^{-1}, \tag{22}$$

where

$$\begin{aligned} \hat{A} &= -\frac{1}{T}\sum_{i=1}^{T}\frac{\partial^2 \log l_t(\hat{\theta})}{\partial\theta\partial\theta'}, \\ \hat{B} &= \frac{1}{T}\sum_{i=1}^{T}\frac{\partial \log l_t(\hat{\theta})}{\partial\theta}\frac{\partial \log l_t(\hat{\theta})'}{\partial\theta}. \end{aligned}$$

In order to minimize the concern of numerical instability in the calculation of second derivatives, we estimate "smoothed" versions of $\hat{A}$ and $\hat{B}$ which are calculated as follows. The $\Delta\theta_1$ and $\Delta\theta_2$ vectors leading to the most stable calculation of first and second derivatives are found. $\hat{A}$ is found by calculating $\hat{A}_i$ using $(0.8 + 0.02i)\Delta\theta_1$, $i = 1, ..., 20$, and letting $\hat{A} = E(\hat{A}_i)$. $\hat{B}$ is found by calculating $\hat{B}_i$ using $(0.8 + 0.02i)\Delta\theta_1$ and $(0.8 + 0.02i)\Delta\theta_2$, $i = 1, ..., 20$, and letting $\hat{B} = E(\hat{B}_i)$. Standard errors using the smoothed estimates and the formula (22) are reported in the first row after parameter estimates. In the second row after parameter estimates we report standard errors using the less theoretically but numerically more robust estimator of the variance-covariance matrix,

$$\hat{\Sigma}_2 = \frac{1}{T}[\frac{1}{T}\sum_{i=1}^{T}\frac{\partial \log l_t(\hat{\theta})}{\partial\theta}\frac{\partial \log l_t(\hat{\theta})'}{\partial\theta}]^{-1} = [T\hat{B}]^{-1},$$

where the smoothed estimate of $\hat{B}$ is used.

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Figures

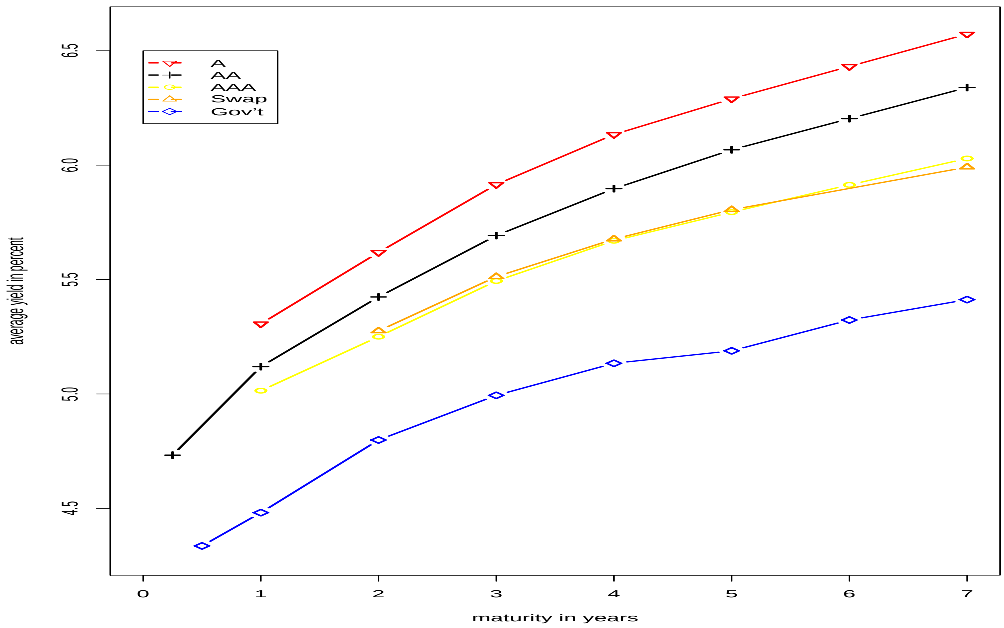

Figure 1: This figure shows the average term structure of swap rates, Treasury yields, and A-, AA-, AAA-corporate yields. Data is from Bloomberg and covers the period December 20, 1996, to February 14, 2003. The Treasury and corporate yields are semi-annual par yields.

Figure 2: This figure shows the 5-year AA, BBB and government par yield and the swap rate. Data is from Bloomberg and covers the period December 20, 1996, to February 14, 2003.

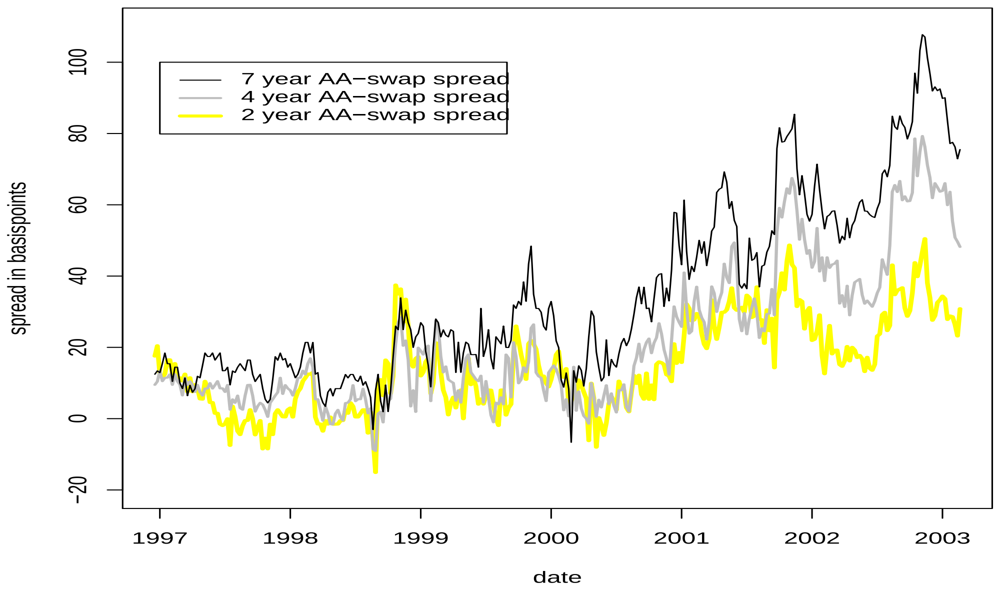

Figure 3: This figure shows the spread between the 2-, 4-, and 7-year AA par yield and swap rate. Data is from Bloomberg and covers the period December 20, 1996, to February 14, 2003.

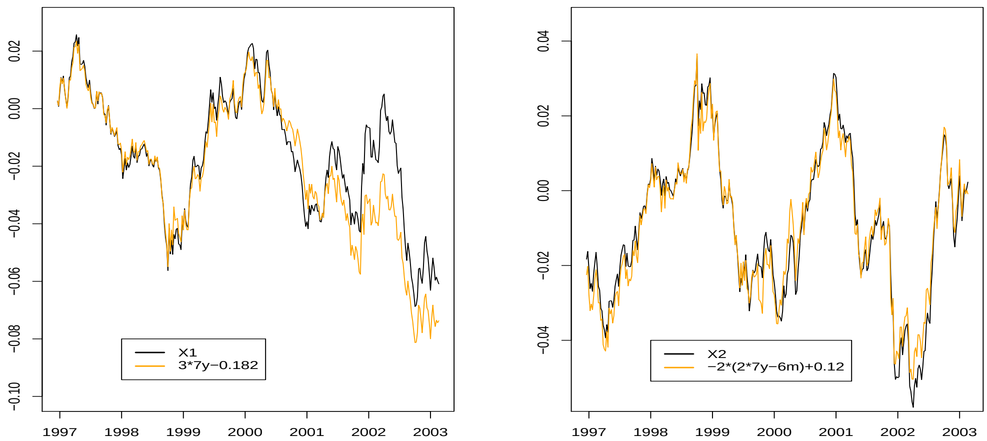

Figure 4: The government rate is given as $r^g(X) = a + X_1 + X_2$ in the model. This figure shows X_1 and X_2 plotted with a function of the level and slope of the government yield curve, and we see that X_1 and X_2 can be interpreted as the level and slope of the government yield curve.

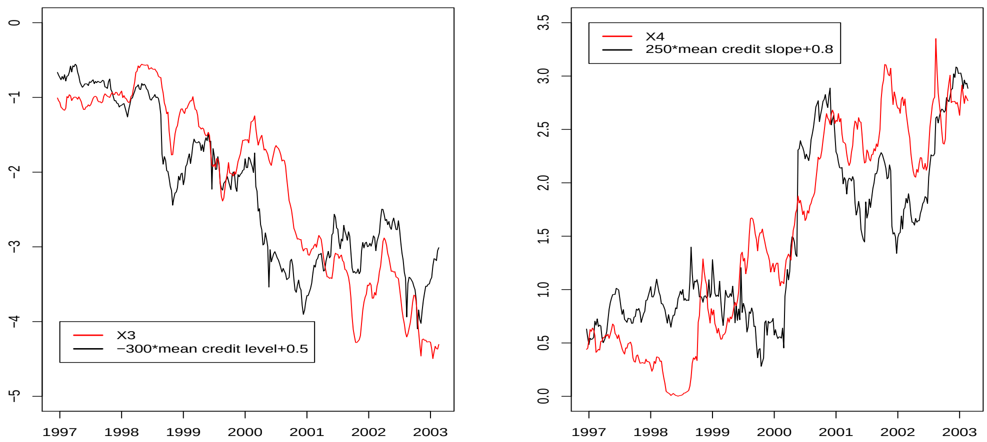

Figure 5: The factor associated with credit risk is given as $\mu(X) = b + X_3 + X_4 + c(X_1 + X_2)$ in the model. This figure shows a) X_3 plotted with a function of the level of the corporate spread curves calculated as $\frac{y_7^{AAA}+y_7^{AA}+y_7^{A}}{3} - y_7^g$, b) X_4 plotted with a function of the steepness of the corporate spread curves calculated as $\frac{y_7^{AAA}+y_7^{AA}+y_7^{A}}{3} - y_7^g - [\frac{y_1^{AAA}+y_1^{AA}+y_1^{A}}{3} - y_1^g]$. The two factors can be interpreted as the level and slope of the credit spread curves although the relationship is not precise due to reasons explained in the text.

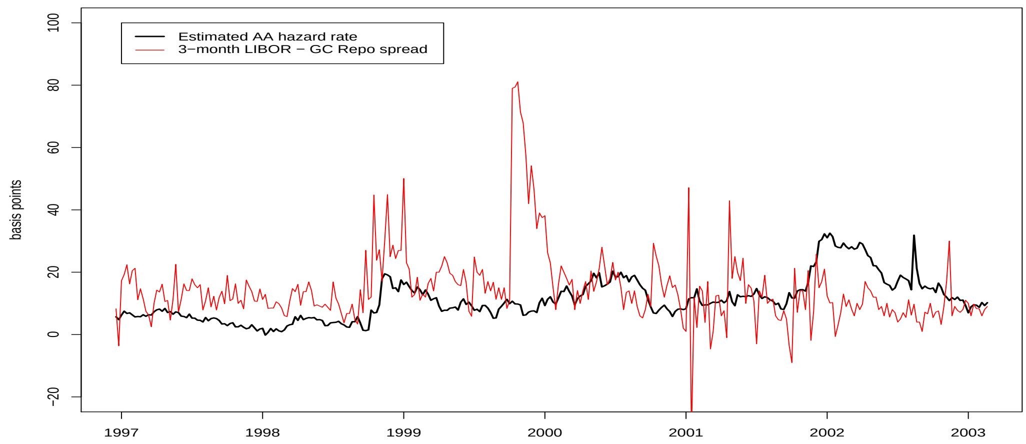

Figure 6: This figure shows the estimated 3-month AA credit risk premium and 3-month LIBOR-GC Repo spread (called LGC spread). The 3-month AA credit risk premium on date t is calculated as the difference in basis points between the yield on a 3-month AA corporate bond and a 3-month riskless bond (with no liquidity). The 3-month LIBOR and GC repo rates are from Bloomberg. The average estimated 3-month AA credit risk premium is 10.0 basis points, while the average LGC spread is 14.7 basis points. Excluding the last three months of 1999 and 2000, the average LGC spread is 12.2 basis points.

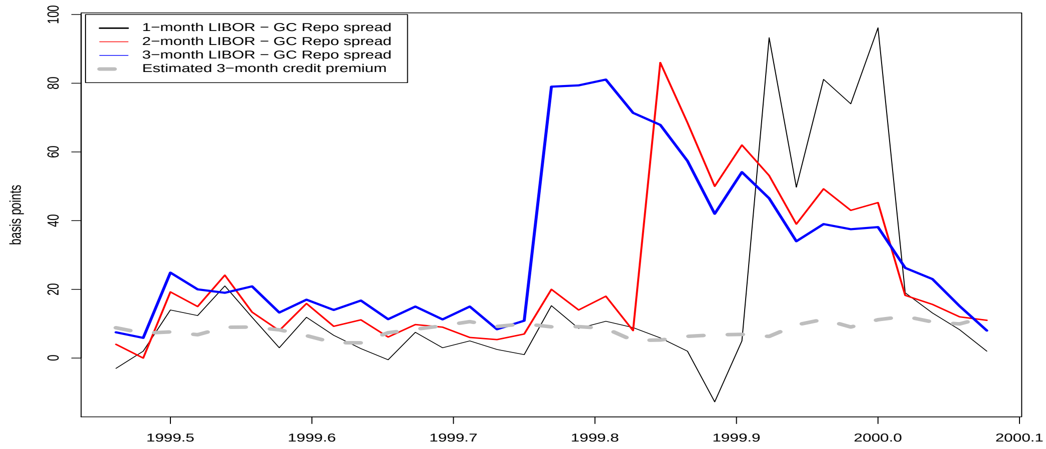

Figure 7: Before Y2K: This figure shows the 1-,2-, and 3-month LIBOR - GC repo spreads along with estimated 3-month Treasury and credit risk premia. The 3-month AA credit risk premium on date t is calculated as the difference in basis points between the yield on a 3-month AA corporate bond and a 3-month riskless bond (with no liquidity). The 3-month Treasury premium on date t is calculated as the difference in basis points between the yield on a 3-month riskless bond (with no liquidity) and a 3-month government bond according to formula (14). LIBOR and GC repo rates are from Bloomberg.

Figure 8: This figure shows the estimated swap factor in the swap market. We relax the assumption of *homogeneous LIBOR-swap market credit quality* and the swap factor accounts for differences in the two markets such as default scenarios, recovery rates, and liquidity.

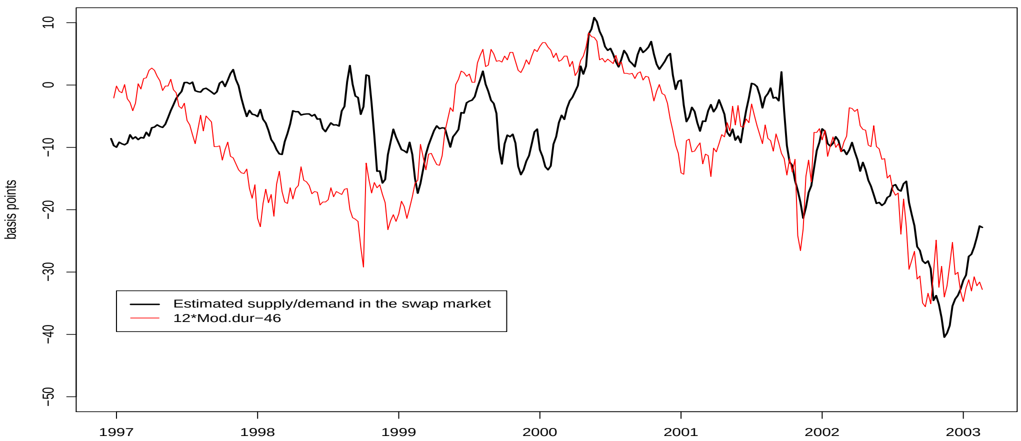

Figure 9: This figure shows the estimated swap factor in the swap market and a linear function of the Lehman option-adjusted duration index for mortgage backed securities downloaded from Datastream. Before Y2000 there is a correlation of -0.01 while after Y2000 the correlation is 0.86.

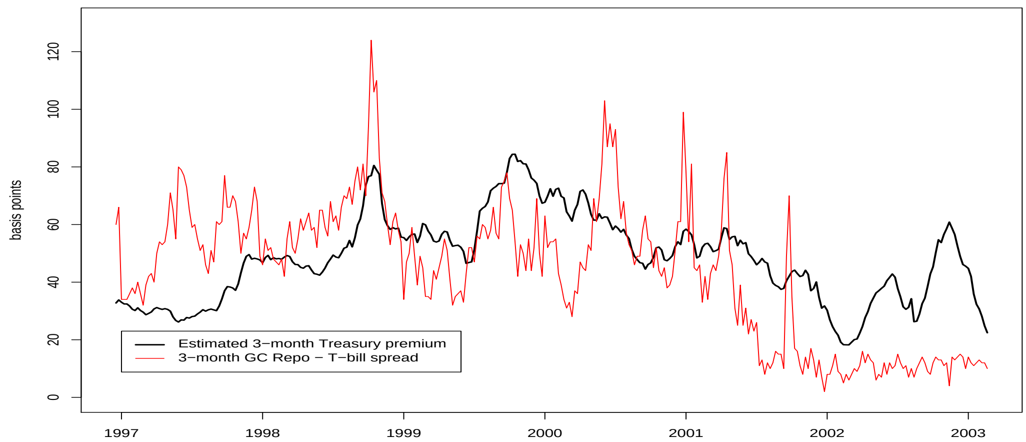

Figure 10: This figure shows the estimated 3-month Treasury premium and the 3-month GC repo-bill spread in the estimation period. 3-month estimated liquidity is calculated according to formula (14) as $-\frac{1}{0.25}\log(E_t(\exp(-\int_t^{t+0.25} L(s)ds))$. The 3-month GC repo is collected from Bloomberg and the 3-month Treasury bill is collected from the Federal Reserve.

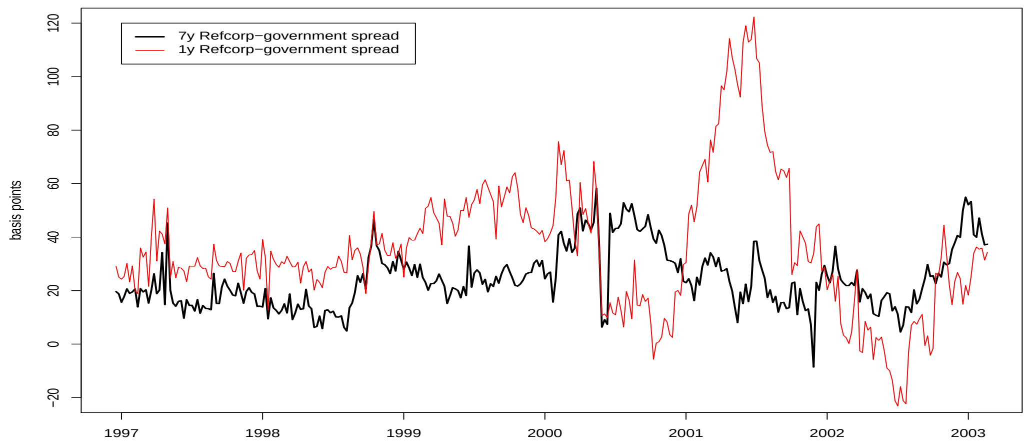

Figure 11: This figure shows the 1- and 7-year spread between Refcorp and government zero coupon yields. Both Refcorp and government yields are zero coupon yields from Bloomberg and Refcorp yields are estimated by Bloomberg using stripped zero coupon yields from Refcorp bonds.

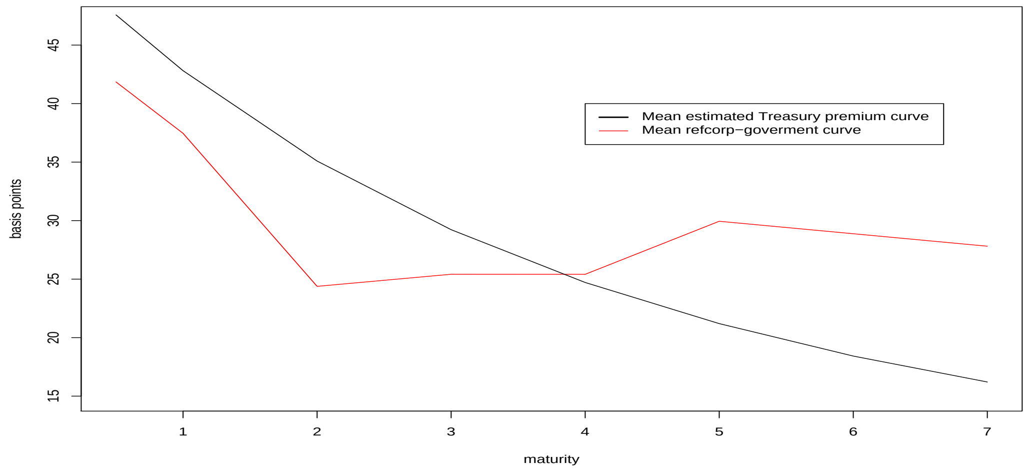

Figure 12: This figure shows the average Refcorp - government spread and average estimated liquidity for maturities 0.5 to 7 years. The average is taken over the estimation period December 20, 1996 to February 14, 2003. Both Refcorp and government yields are zero coupon yields from Bloomberg and Refcorp yields are estimated by Bloomberg using stripped zero coupon yields from Refcorp bonds.

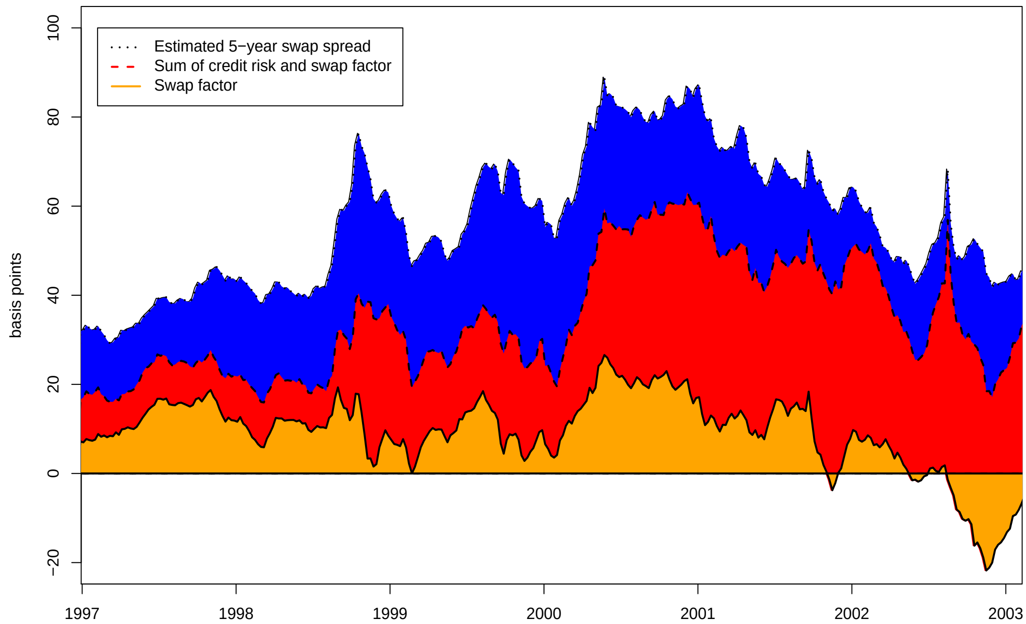

Figure 13: This figure shows a decomposition in basis points of the estimated 5-year swap spread into a swap factor, credit risk factor, and Treasury factor through the estimation period. The size of the swap factor at time t is calculated as $-\frac{1}{5}\log(E_t(\exp(-\int_t^{t+5} S(u)du))$. The size of the Treasury factor at time t is calculated as $-\frac{1}{5}\log(E_t(\exp(-\int_t^{t+5} L(u)du))$. The size of the credit risk factor at time t is calculated as the difference between the estimated 5-year swap spread and the sum of the Treasury and swap factor at time t. The effects are consequently transformed to basis points in par rates. The solid line shows the size of the swap factor. The dashed line shows the sum of the swap and credit risk factor.

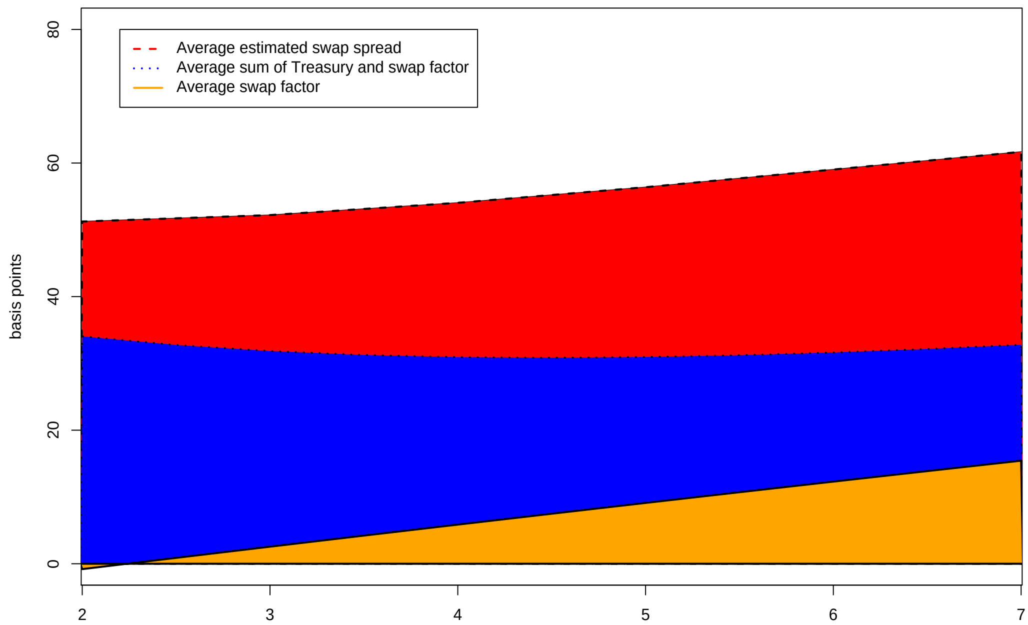

Figure 14: This figure shows the average effect across maturity of the swap factor, credit risk factor, and Treasury factor on swap spreads in the estimation period. The size of the swap factor in swap spreads at maturity T at time t on is calculated as $-\frac{1}{T}\log(E_t(\exp(-\int_t^{t+T} S(u)du))$ and transformed to basis points in par rates. Average is then calculated over the estimation period December 20, 1996 to February 14, 2003. The average effect of the Treasury factor and the credit risk factor is calculated similarly (see caption of Figure 13). The solid line shows the size of the swap factor. The dashed line shows the sum of the swap and credit risk factor.

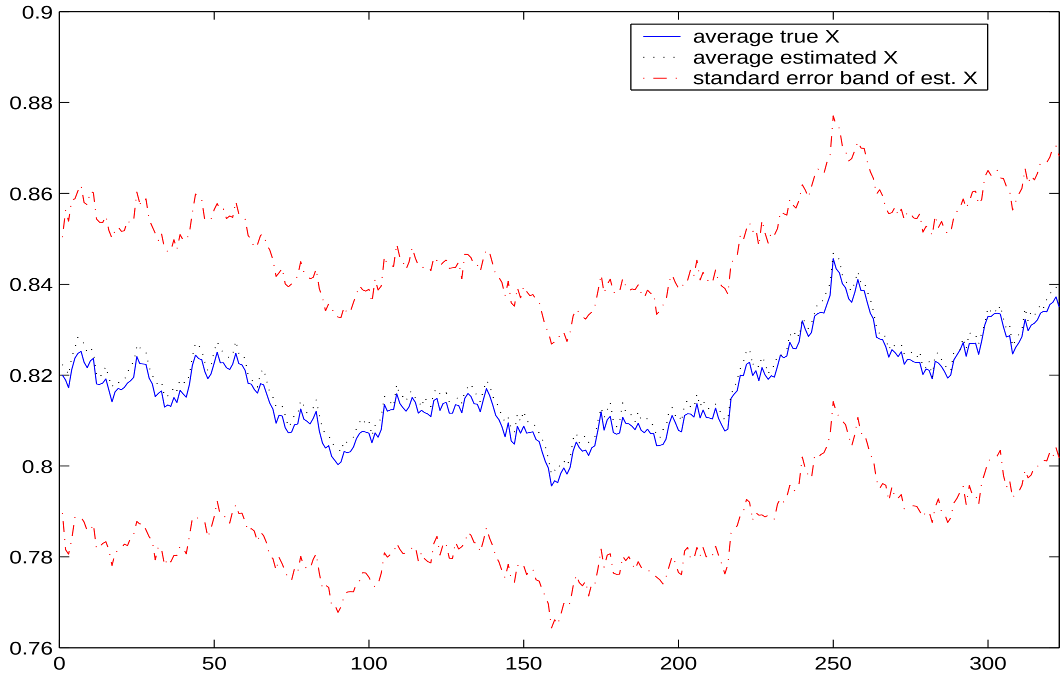

Figure 15: Results of a Monte Carlo experiment for the QML estimator of a one-factor Vasicek model. Average true X at time i, $i = 0, .., 323$ is the mean of X_i in the 500 simulations. Average estimated X at time i, $i = 0, .., 323$ is the mean of the estimated X_i(denoted $\hat{X}_i$) in the 500 simulations. Standard error bands of $\hat{X}_i$ is the standard error of $\hat{X}_i$ in the 500 simulations.