Discussion of:
Correlated Defaults and the Valuation of Defaultable Securities

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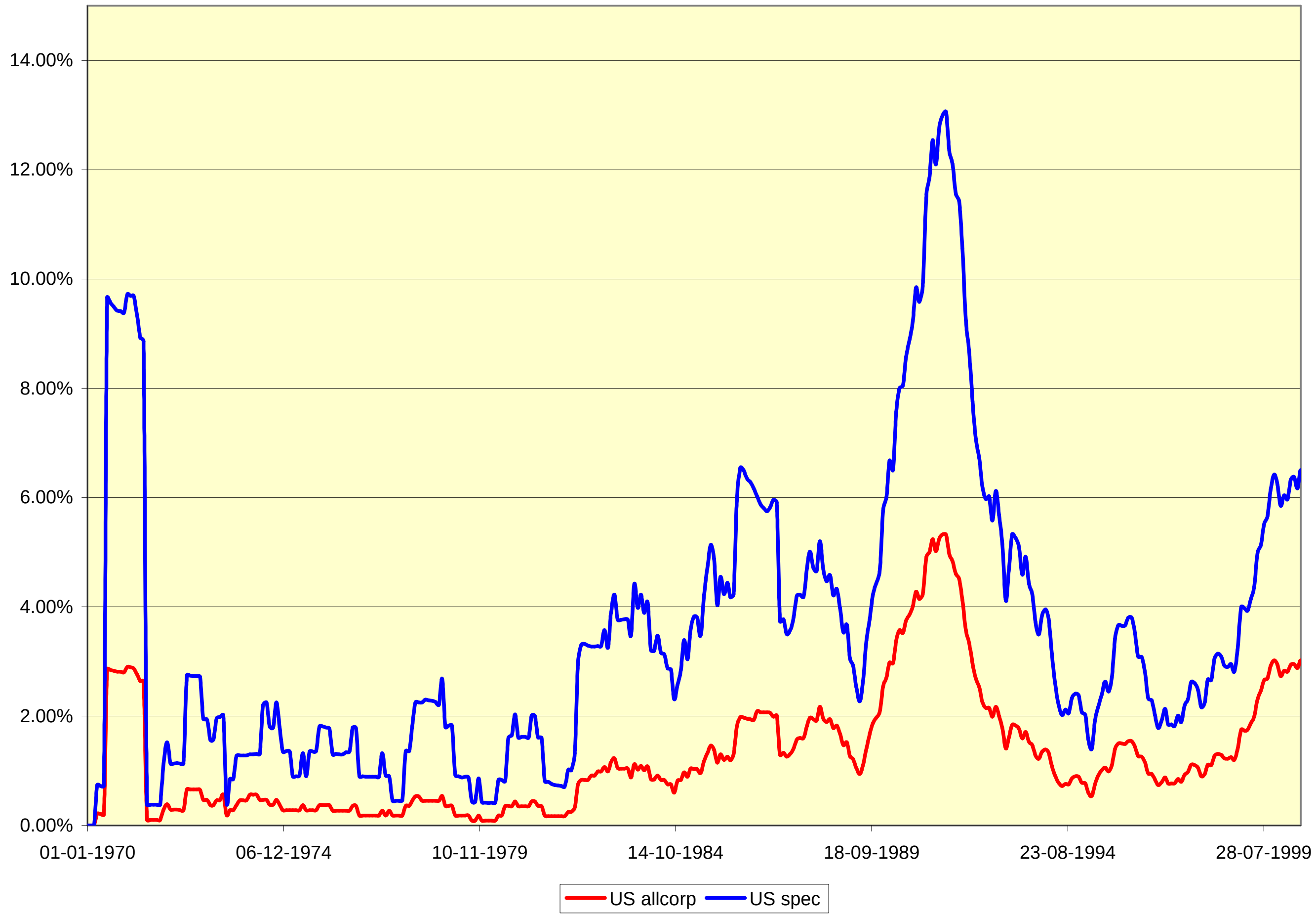
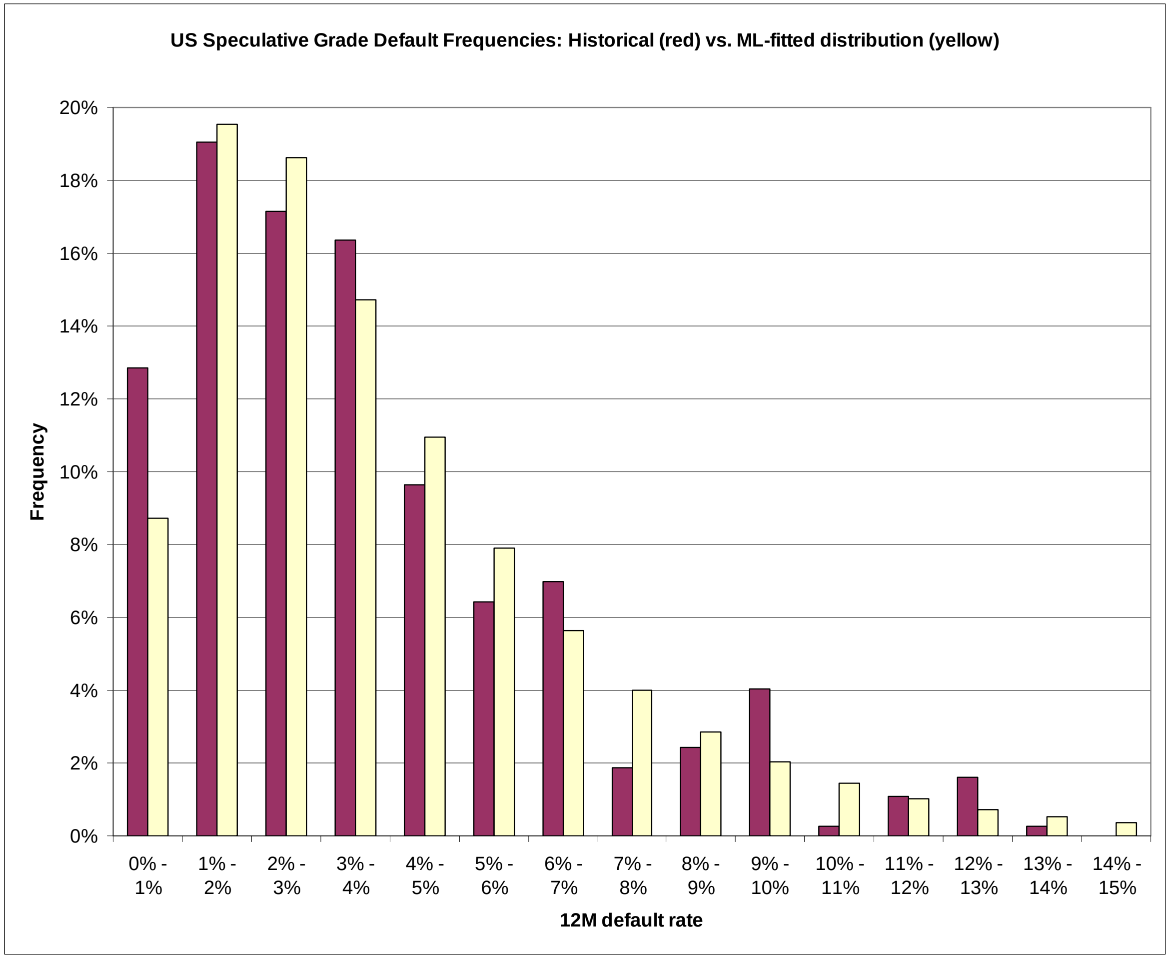
Moody's / Salomon Center Conference on Credit Risk
New York, May 2004

The Problem

Default Dependency: It Exists



12M trailing average default rates of Moody's rated obligors


US Speculative Grade Default Frequencies: Historical (red) vs. ML-fitted distribution (yellow)
Frequency
20%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
0% - 1%
1% - 2%
2% - 3%
3% - 4%
4% - 5%
5% - 6%
6% - 7%
7% - 8%
8% - 9%
9% - 10%
10% - 11%
11% - 12%
12% - 13%
13% - 14%
14% - 15%
12M default rate

Default Dependency: It Matters

- Portfolio credit risk management:
  Multiple defaults are what can sink a financial institution

- Portfolio credit derivatives: CDOs, Basket CDS:
  Multiple defaults drive payoffs.

- Banking supervision and financial stability.

Default Dependency: It is Difficult

- Available Data:
  - ⋆ Multiple defaults are *rare*:
    (by definition) even much rarer than individual defaults
  - ⋆ Dependency information is rare (and often unreliable):
    equity correlations: large estimation errors, other information: even worse
  - ⋆ Market data: Not really liquid enough, yet.
  - ⋆ Only have decent data on *individual* default risk,
    and (typically) something like default rates for fixed time horizons (1Y).
- Modelling Problems:
  - ⋆ Interaction effects
  - ⋆ High dimensionality
  - ⋆ High-level mathematics (hard to avoid jump-processes)

Correlated Defaults and the Valuation of Defaultable Securities

Contribution of the Paper

- Application of the *total hazard* (TH) construction to simulate dependent default times for given default intensity processes.

- Analysis of the algorithm and comparison to other models
  - ⋆ conditional independence
  - ⋆ Copula models

- Proposes some concrete specifications of the model.

- Standard CDS, counterparty risk and basket CDS in this framework.

The General Model Setup and Algorithm

Advantages:

Given (almost) almost **any** specification of default intensities . . .

- . . . following the TH algorithm one can generate samples of times of joint defaults. No numerical difficulties are expected.
- The model specification has an immediate interpretation in terms of the dynamics of the local default probabilities.
- Direct estimation of intensities may be possible using compensator-methods (e.g. Nelson/Ahlen, see Lando/Huge)

As this works in such generality, TH is a modelling *framework* rather than a concrete model. Thus, the key question is the **specification.**

Potential Problems:

- Difficulties to calculate some key quantities necessary *interpret* the model:
  - ⋆ marginal (individual) default probabilities (they are as difficult to evaluate as any joint default event)
  - ⋆ resulting loss distributions for fixed time-horizons
  - ⋆ more complex dynamics (e.g. of longer-horizon CDS or PDs)

- Therefore, the model might also be difficult to . . .
  - ⋆ . . . calibrate to single-name CDS spreads or connect to marginal PDs
  - ⋆ . . . estimate with certain time-horizon datasets

- One can also very easily run into a combinatorical explosion of parameters when enumerating all possible default scenarios.

These problems can be avoided by careful choice of the model's specification.

General Problems

(not restricted to this model)

- "When to stop?"
  If there are cross-influences across all (or most) of the obligors, then we should have to model the whole economy, even if we are just pricing a single-name product.
- Cannot handle **joint** defaults at exactly the same time, as e.g. in Duffie/Singleton (1999b).

Connection to Copula Models

- Copula models can be viewed as intensity-based models. The intensities are given in SS (2001).
- Thus, the TH algorithm also applies to copula models. For intensities which only depend on the default history, they are even equivalent.
- If stochastic intensities are used, copula models can also be more restrictive. (Even though you can also have stochastic intensities in copula models.)
- Big advantage of copula models:
  - ⋆ Immediate calibration to the marginal survival probabilities
  - ⋆ It usually is easy to find a specification which reduces to a given fixed time-horizon (e.g. 1Y) model.
- But copulae also have their disadvantages.

The Concrete Specification and CDS / Basket Pricing

Concrete Specification (used for Baskets)

$$\lambda^i(t) = a + bF(t) + \delta \mathbf{1}_{\{\tau^{(1)} \leq t\}}$$

- The **first** default ($\tau^{(1)}$) has a knock-on effect on other obligors.
- After that, the other obligors are **independent.**
- This can be unrealistic in some cases: Larger portfolios ($I > 5$), low-quality obligors, or long time-horizons.
- In this setup, many calculations can still be performed in closed-form by conditioning on the time of the first default.

Yu gives also a more general specification with a matrix of cross-default influences.
In the most general specification such a matrix itself would be stochastic and depend on the default-history.

First-Default Survival Probabilities

The first default $N^{(1)}(t)$ event has the intensity

$$\sum_{i=1}^{I} \lambda^i(t)\mathbf{1}_{\{\tau^{(1)}>t\}} = I(a + bF(t))\mathbf{1}_{\{\tau^{(1)}>t\}}$$

Thus, the FtD survival function is:

$$\overline{P}^{(1)}(T) = \mathbf{P}\left[\ \tau^{(1)} > T\ \right] = e^{-IaT}\mathbf{E}\left[\ e^{-Ib\int_0^T F(t)dt}\ \right].$$

The density of $\tau^{(1)}$ is then easily reached by differentiation. Let's call it $p^{(1)}(t)$.

Individual Survival Probabilities

The trick is to split the event up in two sub-events: one where $\tau^{(1)} \leq T$ and one where $\tau^{(1)} > T$:

$$\begin{aligned}\overline{P}_i(T) = \mathbf{P}\left[\, \tau_i > T \,\right] =& \mathbf{P}\left[\tau_i > T \,\middle|\, \tau^{(1)} > T\right] \mathbf{P}\left[\,\tau^{(1)} > T\,\right] \\ &+ \mathbf{P}\left[\tau_i > T \,\middle|\, \tau^{(1)} \leq T\right] \mathbf{P}\left[\,\tau^{(1)} \leq T\,\right] \\ =& \overline{P}^{(1)}(T) + \int_0^T \mathbf{P}\left[\tau_i > T \,\middle|\, \tau^{(1)} = t\right] p(t)dt,\end{aligned}$$

and of course $\mathbf{P}\left[\,\tau_i > T \,\middle|\, \tau^{(1)} = t\,\right]$ is easily calculated.

In the same manner, other quantities, e.g. the probability of $n > 1$ defaults can be calculated. (Using that defaults are **conditionally independent** after $\tau^{(1)}$.) There may be problems with stochastic $F(t)$ for higher-order defaults.

Counterparty Risk and CDS Pricing

- Some of the simplifications in the CDS specification (e.g. non-defaultable fee streams) are not necessary.
- Specification of the counterparty risk in CDS: If the fee payer A defaults (stopped paying at τ_A), then
  - ⋆ I would **not** expect the protection seller to make the protection payment if a credit event occurs later on.
  - ⋆ The CDS would have to be unwound, either at market value (if it has a positive value to A), or at market value times recovery rate of A (if it has a negative value to A).

Summary

- TH algorithm seems an efficient and feasible numerical procedure for the simulation of general intensity models (beyond multivariate Cox processes).
- Work needs to be done regarding model specification and estimation.
  - ⋆ "contagion" idea is attractive
  - ⋆ challenge is to find a specification which avoids the "curse of dimensionality" for larger number of obligors
  - ⋆ ... yet is should still be realistic, easily parametrised, and it should capture dependency for multiple defaults
- Unclear whether an advantage is reached for *analytical* purposes (i.e. pencil-and-paper calculations).