Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds

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1 Introduction

- Structural models fail to explain level of yield spreads on corporate bonds
  - e.g. Huang and Huang (2003):

	Model	Actual data
A	23	123
BBB	56	194

- Two possible explanations for failure of structural models:
  - failure to predict accurately probability of default/recovery rate
  - effect of factors other than credit or interest rates

1.1 Introduction: What we know about Structural Models

- Structural Models: A Puzzle
  - do not explain level of prices / credit spreads
  - BUT .. appear to provide reasonably good estimates of default probabilities (Leland 2002), KMV

- This paper: Focus is on
  - hedge ratios/*price sensitivity* ($second\text{-}moment$ predictions of models)
  - rather than *level* of price/credit spread (*first moment* predictions)

- Why is this important?
  - in structural models the bond value is determined as the price of the replicating portfolio
  - composition of the replicating portfolio is determined by the hedge ratios

1.2 Main Finding

- Simple structural model (Merton, 1974) provides reasonably good estimates of hedge ratios of corporate debt to equity

- BUT returns on corporate bonds also strongly related to:
  - VIX .. but NOT via credit risk channel
  - SMB (FF factor) but not in way that is related to exposure to underlying equity

- A new Puzzle: why are these factors present and do they affect pricing?

2 Data

- Merrill Lynch Corporate Master Index and Corporate High Yield index
  - Covers nearly all corporate bond issues in the U.S. (2800 issuers; 8700 issues)

- Monthly price data from 12.1996–09.2002 (308,000 bond-month observations)

- Final sample satisfies additional standard criteria (only US bonds, matching with CRSP/COMPUSTAT, no financials)
  - Final sample (descriptive statistics in Tables I, II and III):

	Time-Series	Cross-Section
Bonds	1595	3631
Issuers	536	1047

Estimated Leverage and Volatilities

	All	AAA	AA	A	BBB	BB	B	CCC
	Quasi-Market Leverage							
Mean	0.36	0.04	0.15	0.24	0.38	0.47	0.60	0.75
Std. Dev.	0.23	0.11	0.13	0.15	0.20	0.20	0.22	0.20
	Equity Volatility							
Mean	0.37	0.26	0.25	0.29	0.34	0.44	0.57	0.84
Std. Dev.	0.18	0.05	0.07	0.09	0.11	0.13	0.22	0.35
	Estimated Asset Volatility							
Mean	0.24	0.25	0.22	0.22	0.22	0.26	0.31	0.38
Std. Dev.	0.10	0.05	0.07	0.08	0.09	0.10	0.13	0.13

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- Quasi-market leverage:

$$\frac{\text{Book value of Debt (COMPUSTAT items 9 and 34)}}{\text{Book value of Debt (9+34)} + \text{Market value of Equity } (216 \times 199)}$$

- Estimated asset volatility:

$$\widehat{\sigma^2_{Ajt}} = (1 - L_{jt})^2 \sigma^2_{Ejt} + {L_{jt}}^2 \sigma^2_{Djt} + 2L_{jt}(1 - L_{jt})\sigma_{ED,jt},$$

2.1 A Simple Time-Series Hedging Regression

- We run the following regression: (Table IV)

$$\overline{r}_{j,t} = \alpha_{j,0} + \alpha_{j,E}\overline{r}_{E,t} + \alpha_{j,rf}\overline{rf}_{10y,t},$$

	All	AAA	AA	A	BBB	BB	B	CCC
Intercept	-0.06	-0.14	-0.11	-0.12	-0.10	0.20	0.15	-0.73
	(-0.93)	(-1.77)	(-2.32)	(-3.45)	(-1.63)	(1.72)	(0.79)	(-1.53)
rf_{10y}^{ret}	44.54	87.51	69.01	60.41	51.97	19.71	-11.59	-35.79
	(14.23)	(21.13)	(27.14)	(32.72)	(15.36)	(3.14)	(-1.11)	(-1.34)
E^{ret}	4.08	-0.20	1.44	2.06	3.78	7.09	9.39	12.29
	(10.09)	(-0.23)	(2.76)	(6.44)	(7.89)	(9.60)	(11.04)	(8.65)
$\bar{R}^2$	0.39	0.71	0.57	0.48	0.36	0.24	0.14	0.28
N	42.43	47.72	48.74	42.75	42.09	41.59	39.39	36.54
	(1595)	(11)	(125)	(553)	(540)	(145)	(208)	(13)

2.2 Are These Hedge Ratios Reasonable?

- Compare with:
  - hedge ratios from cross-sectional regressions
  - hedge ratios implied by Merton model

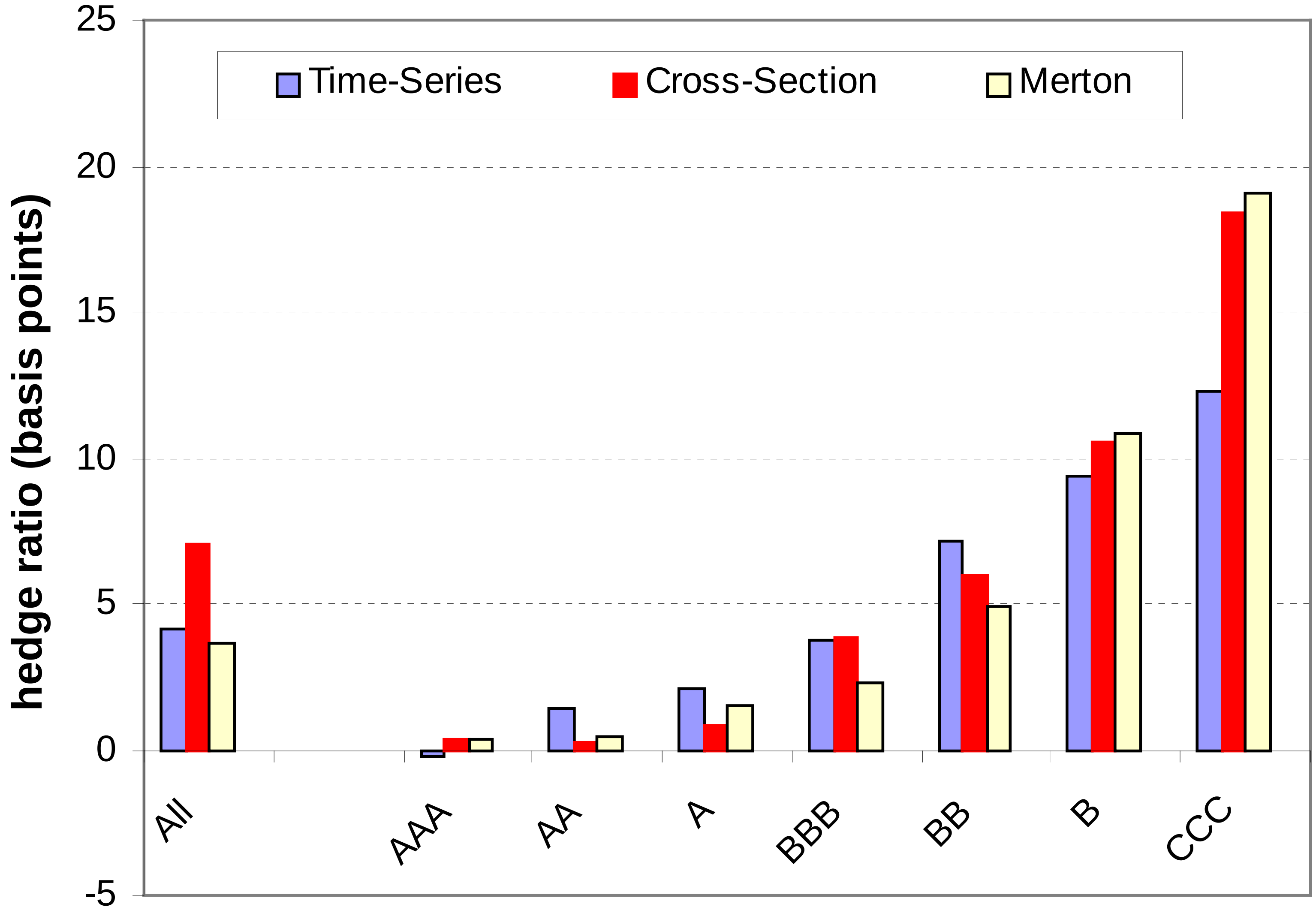
- In one-factor structural models the hedge ratio, h_E, is

$$\frac{\partial D}{\partial E}\frac{E}{D} = \left(\frac{\frac{\partial D}{\partial V}}{\frac{\partial E}{\partial V}}\right)\frac{E}{D} = \left(\frac{1}{\frac{\partial E}{\partial V}} - 1\right)\frac{E}{D},$$

- In the Merton (1974) model

$$h_E = \left(\frac{1}{\Delta_E} - 1\right)\left(\frac{1}{L} - 1\right),$$

where Δ_E is the “delta” of equity against underlying assets of the firm



25
20
15
10
5
0
-5
hedge ratio (basis points)
Time-Series
Cross-Section
Merton
All
AAA
AA
A
BBB
BB
B
CCC

2.3 Testing the Merton Model's Hedge Ratio Predictions

Time-Series regressions: $\overline{r}_{j,t} = \alpha_{j,0} + \beta_{j,E}[h_{E,j,t}\overline{r}_{E,t}] + \alpha_{j,rf}\overline{rf}_{10y,t}$ (Table IV)

	All	AAA	AA	A	BBB	BB	B	CCC
$\overline{\beta_E}$	1.18	-2.91	0.53	0.92	1.61	1.57	0.88	0.60
	(0.30)	(-0.22)	(-0.30)	(-0.10)	(1.01)	(1.04)	(-1.24)	(-5.05)
$\bar{R}^2$	0.34	0.70	0.53	0.43	0.34	0.22	0.16	0.24
N	42.94	49.56	51.40	43.69	42.44	42.39	39.95	36.50
	(1328)	(9)	(70)	(426)	(487)	(132)	(192)	(12)

Cross-Sectional regressions: $\overline{r}_{j,t} = \alpha_0 + \alpha_E h_E \overline{r}_{E,j,t} + u_{j,t}$ (Table V)

	All	AAA	AA	A	BBB	BB	B	CCC
$\overline{\alpha_E}$	0.98	1.13	0.63	0.79	2.03	1.46	0.98	1.01
	(-0.19)	(0.02)	(-0.39)	(-0.66)	(1.80)	(1.78)	(-0.22)	(0.10)
$\bar{R}^2$	0.12	0.12	0.01	0.02	0.05	0.08	0.15	0.17
N	1430.45	8.68	95.55	443.82	537.12	131.43	175.98	37.87
	(60)	(60)	(60)	(60)	(60)	(60)	(60)	(60)

2.4 Story so far

- Merton model produces hedge ratios in line with empirical estimates

- So, structural models appear to capture credit exposure quite well

- However, R^2 are much lower than the model predicts
  - $R^2 \sim 15\% - 25\%$ for non-investment grade bonds
  - $R^2 \sim 40\% - 70\%$ for investment grade bonds

- What other factors influence corporate bond returns?

2.5 Corporate bond returns are sensitive to VIX

- Running a "kitchen sink" regression does not greatly change hedge ratios for riskless rate and equity
- S&P: not significant (related to VIX)
- Sensitivity to VIX highly significant *for investment grade* (c.f. CDGM (2001))

(From Table XII)

	All	AAA	AA	A	BBB	BB	B	CCC
VIX change	-5.6	-5.8	-6.0	-4.7	-5.6	-5.3	-7.3	-11.9
	(-2.96)	(-2.30)	(-4.30)	(-4.41)	(-2.89)	(-1.35)	(-1.08)	(-0.76)

- Natural interpretation: VIX proxies for changes in equity volatility.
  - No: sensitvitiy to VIX very similar across rating categories as well as within different asset volatility and leverage classes
- Preliminary conclusion: impact of VIX is unrelated to credit risk

2.6 Corporate bond returns are also sensitive to SMB

- Sensitivity to SMB even more significant than to VIX

- Sensitivity is NOT related to exposure of bond returns to equity

All	AAA	AA	A	BBB	BB	B	CCC
SMB coefficients from regressions from Table XII							
10.73	6.46	6.69	9.22	10.42	18.28	12.66	14.45
(7.11)	(3.34)	(5.89)	(10.68)	(6.65)	(6.07)	(2.37)	(0.98)
SMB coefficients from regressions on residuals from Table IV							
9.74	6.04	6.10	8.52	9.44	15.90	11.76	11.57
(6.95)	(3.48)	(5.68)	(10.42)	(6.34)	(5.75)	(2.42)	(0.92)

3 Next Steps

- Extend to richer set of structural models and in particular:
  - More reasonable modelling of default boundary
  - Include stochastic riskless interest rates

- Apply this approach to hedging portfolios of corporate bonds