Moody's/NYU inaugural credit risk conference

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Portfolio credit risk - an overview

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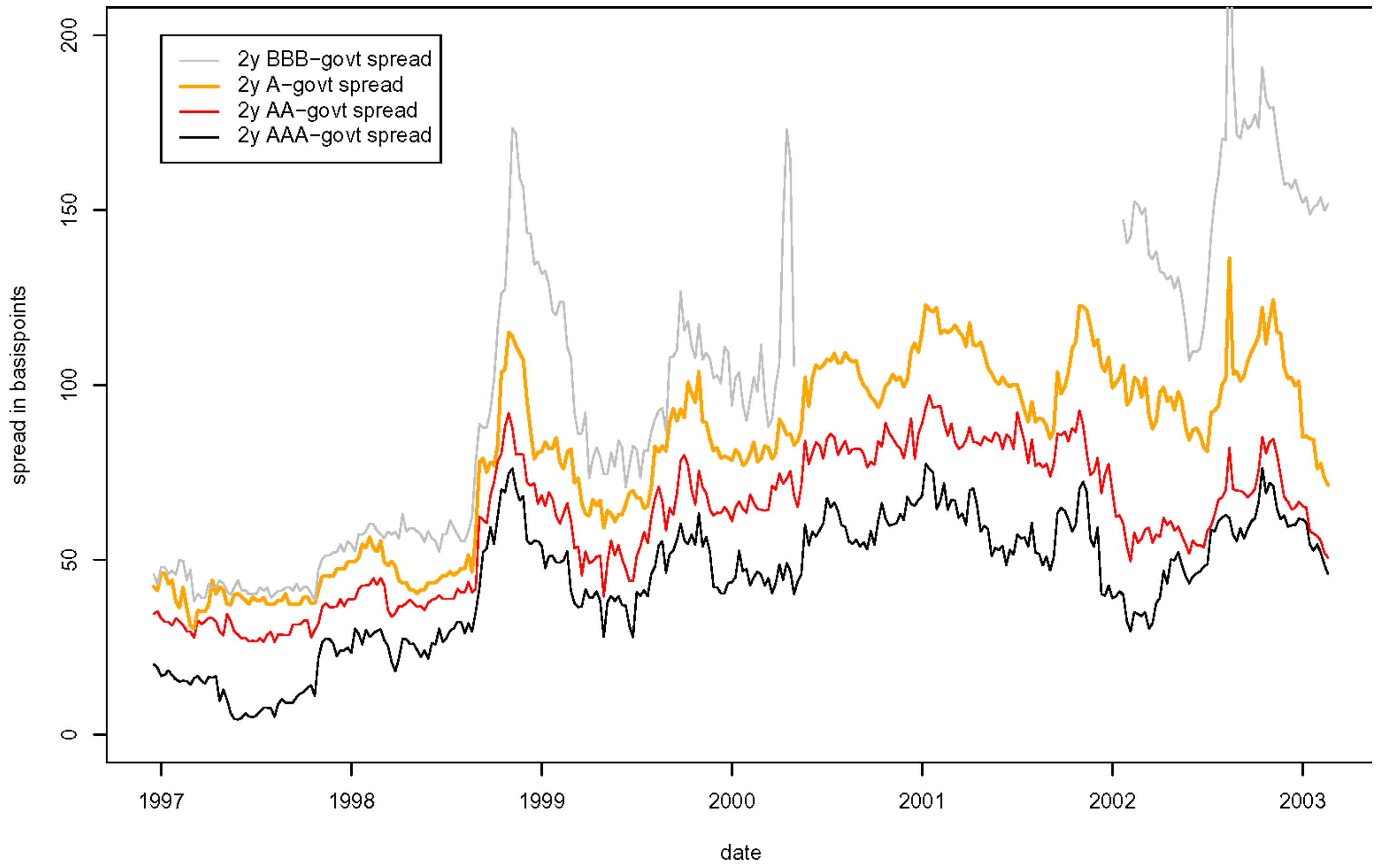
Covariation of credit spreads



The challenges – and some answers

Modeling dependence

- Factor dependence – observable factors (pd's and rec)
- Factor dependence – latent factors (pd's and rec)
- Defaults causing increases in default probabilities
- Defaults causing defaults (contagion)
- Copula functions

Dealing with inhomogeneity

- Homogeneous approximation (diversity scores)
- Moment generating functions
- Tail risk
- Simulation

The mixed binomial distribution

Pure binomial distribution for number of losses D among N issuers when default probability is p:

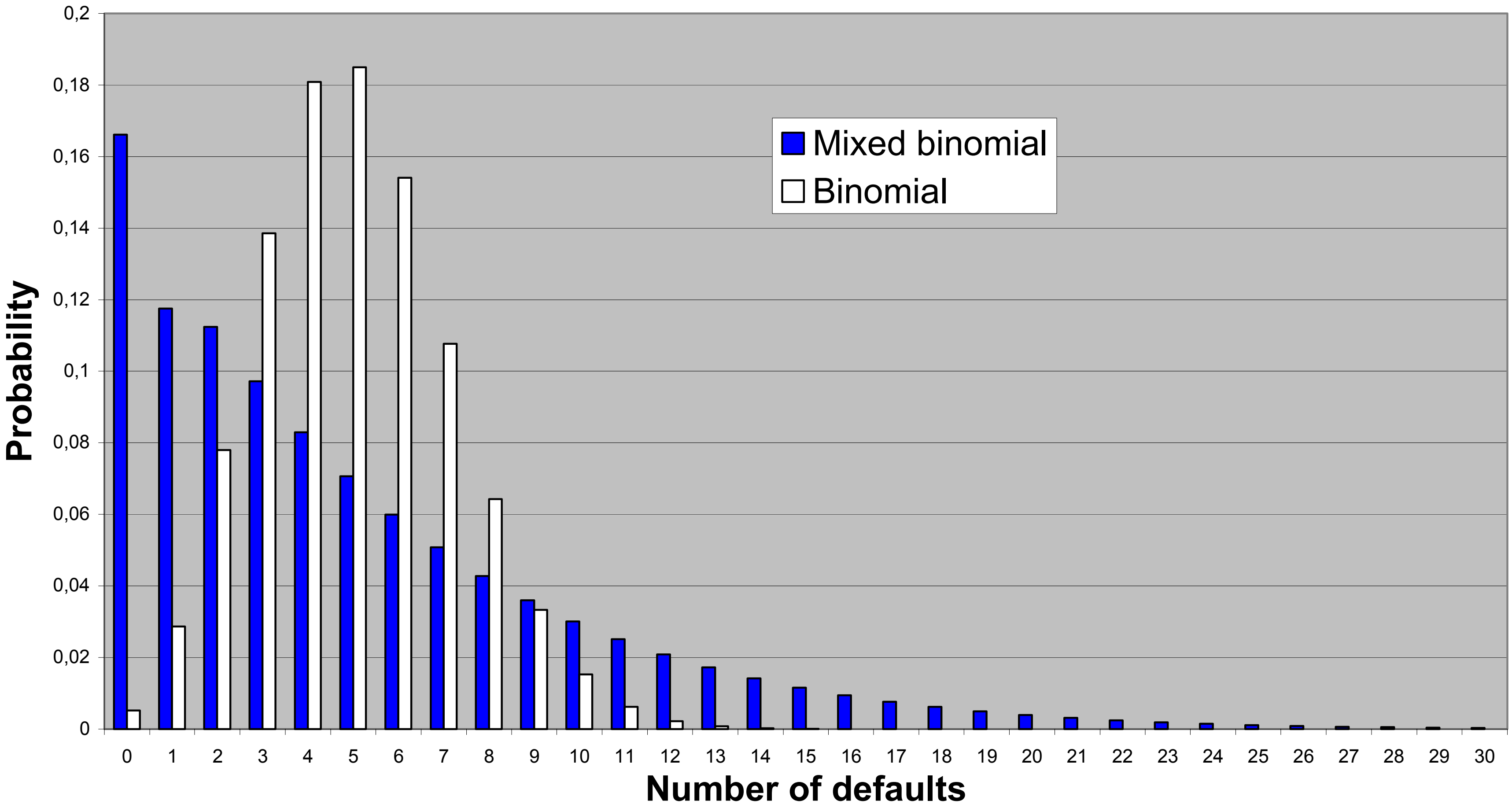
$$\Pr(D = k) = \binom{N}{k} p^k (1 - p)^{N-k}$$

Randomizing over p gives mixture:

$$\Pr(D = k) = \int_0^1 \binom{N}{k} p^k (1 - p)^{N-k} f(p) dp$$

Conditionally on 'factor' p,

the default events are independent



Probability
0,2
0,18
0,16
0,14
0,12
0,1
0,08
0,06
0,04
0,02
0
Mixed binomial
Binomial
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Number of defaults

A mixture distribution arising from Merton's model

$$dV_t^i = \mu V_t^i dt + \sigma V_t^i (\rho W_t^0 + \sqrt{1-\rho^2} W_t^i)$$

$W_t^0 =$ Brownian motion driving factor risk

$W_t^i =$ Brownian motion driving idiosyncratic risk

Default when $V_T < D$

Given outcome of factor, defaults are independent

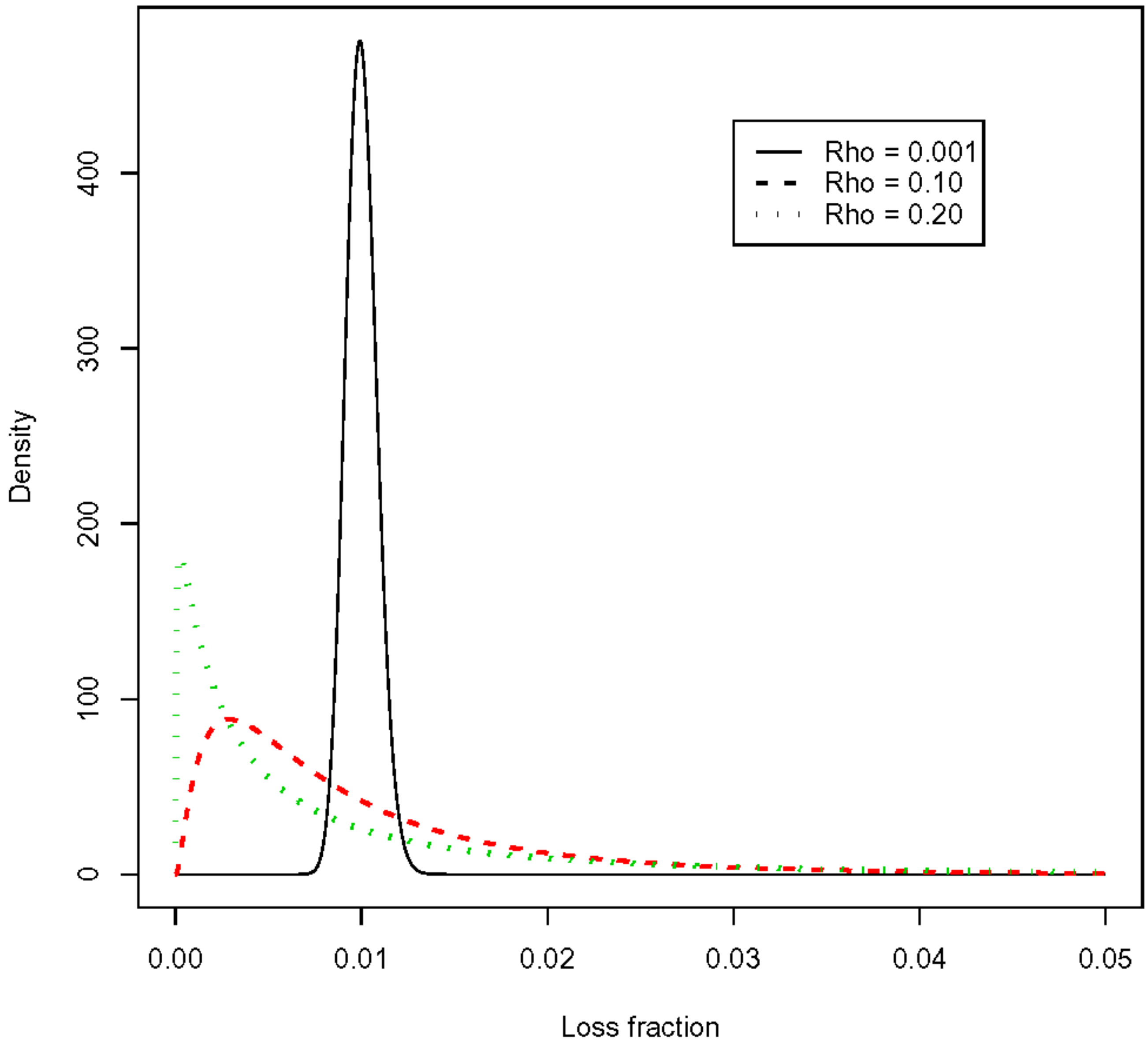
Probability of default given the factor easy to derive

Use distribution of factor to get mixture distribution

$$F(\theta) = N\left(\frac{1}{\rho}\left(\sqrt{1-\rho^2}N^{-1}(\theta) - N^{-1}(\overline{p})\right)\right)$$

$\overline{p} =$ unconditional default probability

Vasicek (1991)



Rho = 0.001
Rho = 0.10
Rho = 0.20
Density
Loss fraction
0
100
200
300
400
0.00
0.01
0.02
0.03
0.04
0.05

The reduced form model analogue

$$\lambda_t^i = \nu_t^c + \nu_t^i$$

The default intensity is a sum of an idiosyncratic component and a factor component

Affine specifications (typically with independence between the two components) gives analytical tractability

To induce ’noticeable’ dependence, a fairly strong variation in common factor is required

Again, homeogeneity gives ’binomial’ type formulas

Duffie and Garleanu (2001)

Gordy (2000)

The parameter μ measures mean size of exponentially distributed jumps occuring with intensity 0.2.

Long run intensity level = 0.02

CIR with jumps

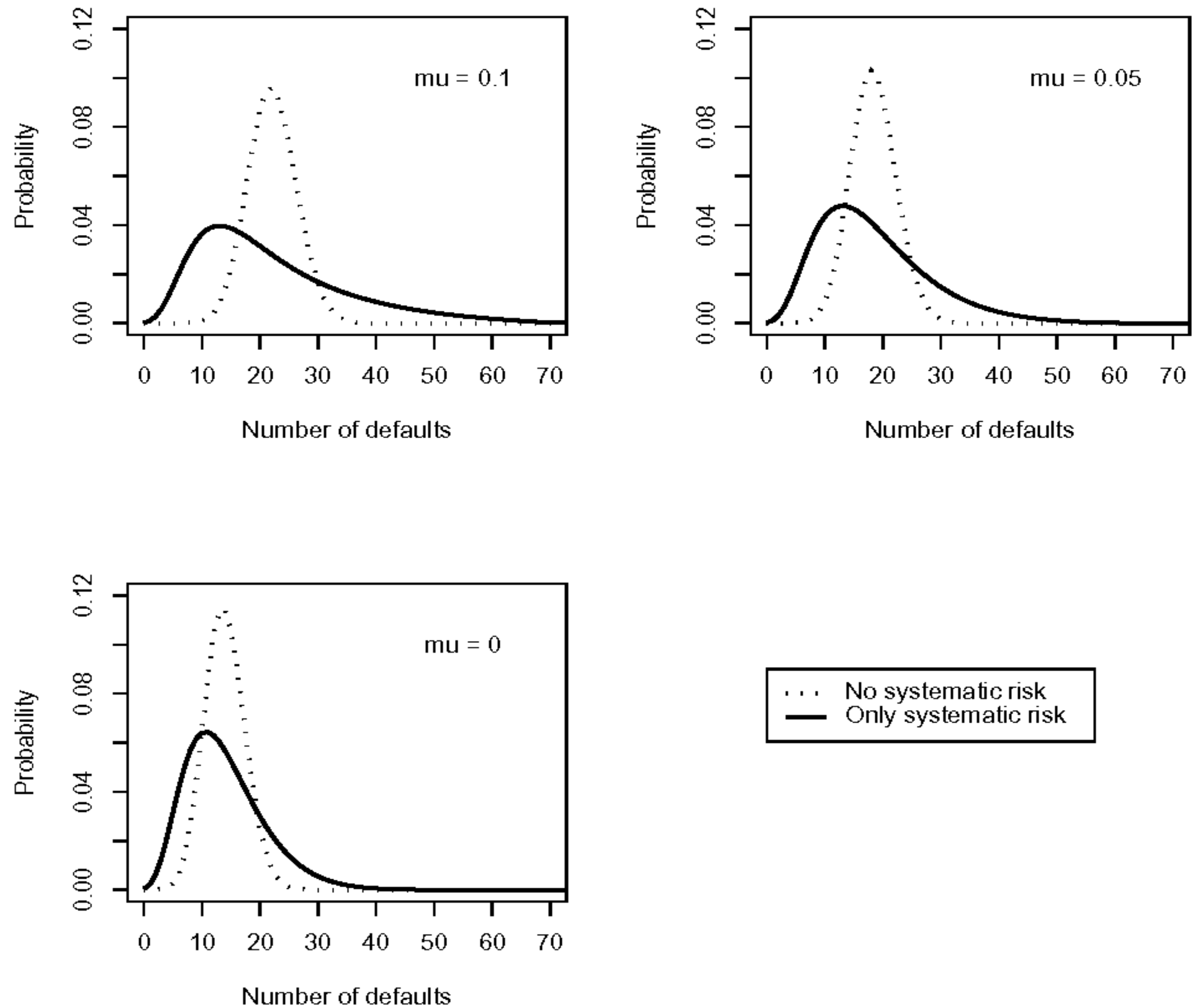
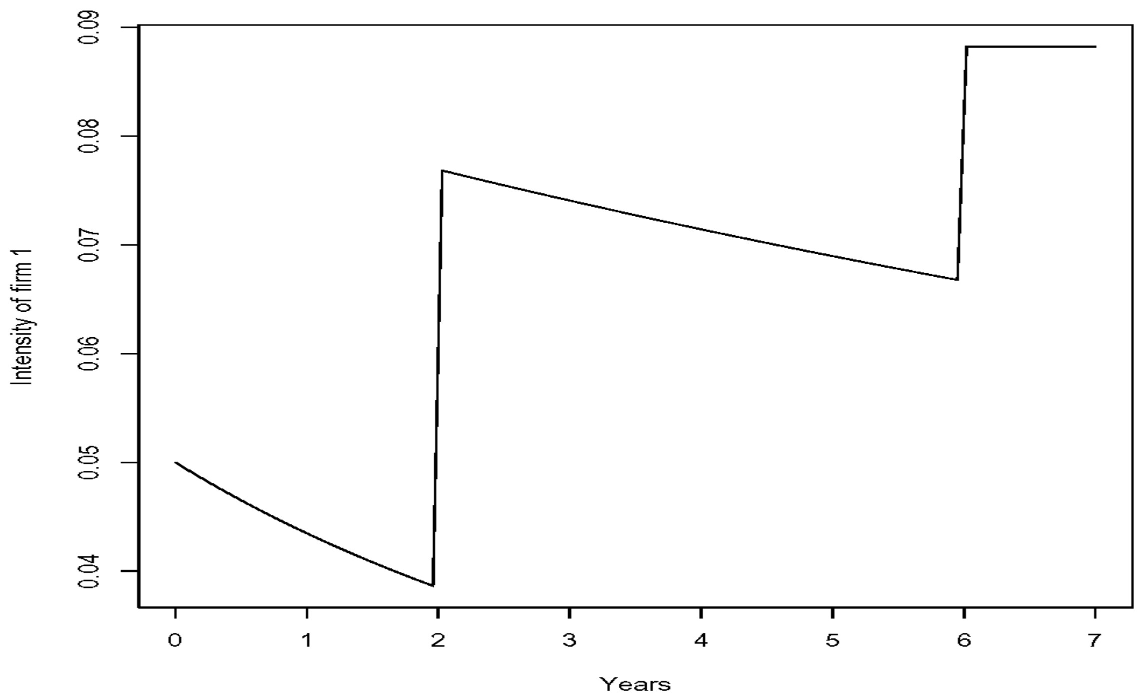


Illustration of latent variable effect



Arjas (1988)

Collin-Dufresne, Goldstein, Helwege (2003)

Intensity of firm 1 is 0.05Z when alive

Intensity of firm 2 is 0.10Z when alive

Z is unobserved exponential with mean 1

We observe firm 2 to default at time 2 and firm 1 at time 6

Contagion

$p =$ probability of direct default p

$q =$ probability that default by i causes default of j

$X_i =$ Indicator for direct default

$Y_{ij} =$ Indicator for default of i 'infecting' j

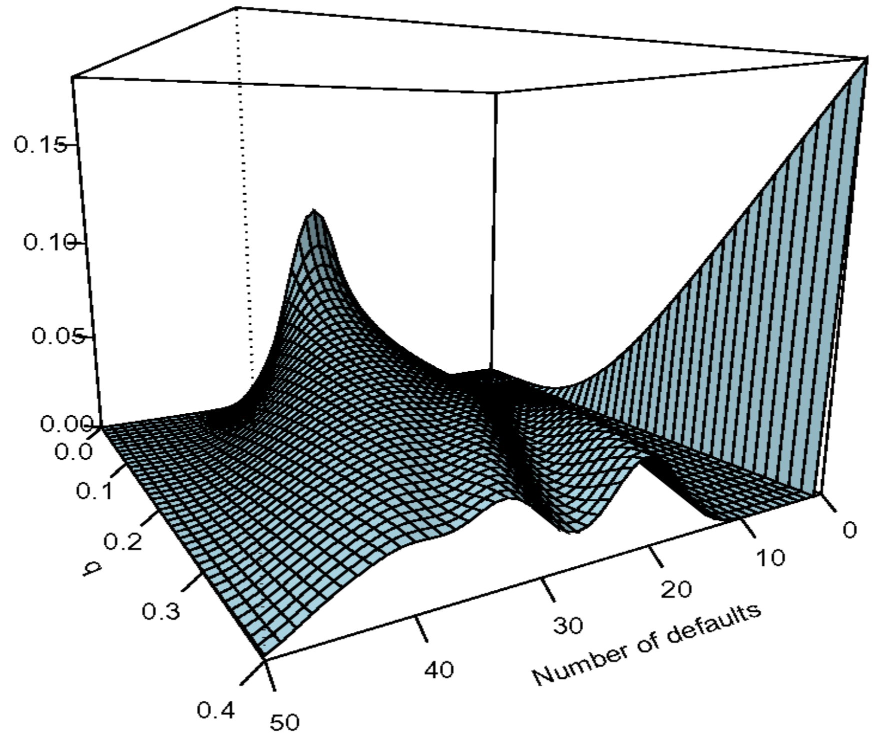
X_i, Y_{ij} independent Bernoulli variables

$\text{Prob}(X_i = 1) = p \quad \text{Prob}(Y_{ij} = 1) = q$

Default indicator becomes

$$\text{Z}_\text{i} = X_i + (1 - X_i)(1 - \prod_{j \neq i} (1 - X_j Y_{ji}))$$

Davis and Lo (2001)



Contagion model with default probability held constant at 0.5 with varying degree of contagion

Contagion between two firms as a continuous-time Markov chain

$$\begin{array}{c} \\ (N,N) \\ (D,N) \\ (N,D) \\ (D,D) \end{array} \begin{array}{c} \begin{array}{cccc} (N,N) & (D,N) & (N,D) & (D,D) \end{array} \\ \begin{pmatrix} -(a_1+a_2-\delta) & a_1-\delta & a_2-\delta & \delta \\ 0 & -b_2 & 0 & b_2 \\ 0 & 0 & -b_1 & b_1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

N=non-defaulted,

D=defaulted

When both are alive intensities are a_1 and b_1

δ governs intensity of joint default (direct contagion)

If $b_1 > a_1$ and $b_2 > a_2$, then default of one

raises the intensity of default of the other.

Solved analytically or by taking matrix exponentials

Works only for few firms

Jarrow and Yu (2001)

Collin-D, Goldstein, Hugonnier (2002)

Lando (1998)

Copulas

- An n-dimensional copula function is a **distribution function** on $[0,1]^n$ with **uniform one-dimensional marginal distributions.**
- Indpendence case: $C(x,y)=xy$
- Corresponding density function is 1 (we think only of cont. case)
- For general copulas, density will be larger than 1 in some areas, smaller than 1 in others.
- If higher than 1 at (say) (0.9,0.9) then distributions are more likely than in the independence case to be jointly around 90 pct quantile.
- Copula functions allow us to 'pair up' marginal distributions to have such dependencies

Schönbucher (2003)

Copulas

- Given one-dimensional distribution functions F_1, F_2 and copula C.
- Define a new distribution function as

$$F(x, y) = C(F_1(x), F_2(y))$$

- Sets up a joint distribution with the right marginal distributions
- Used in practice to simulate dependent data
- Algorithm is copula-specific
- Often difficult to interpret parameters
- Consistency issues

Dealing with non-homogeneity

(different face values, different default probabilities…)

Trying to get back to homogeneity:

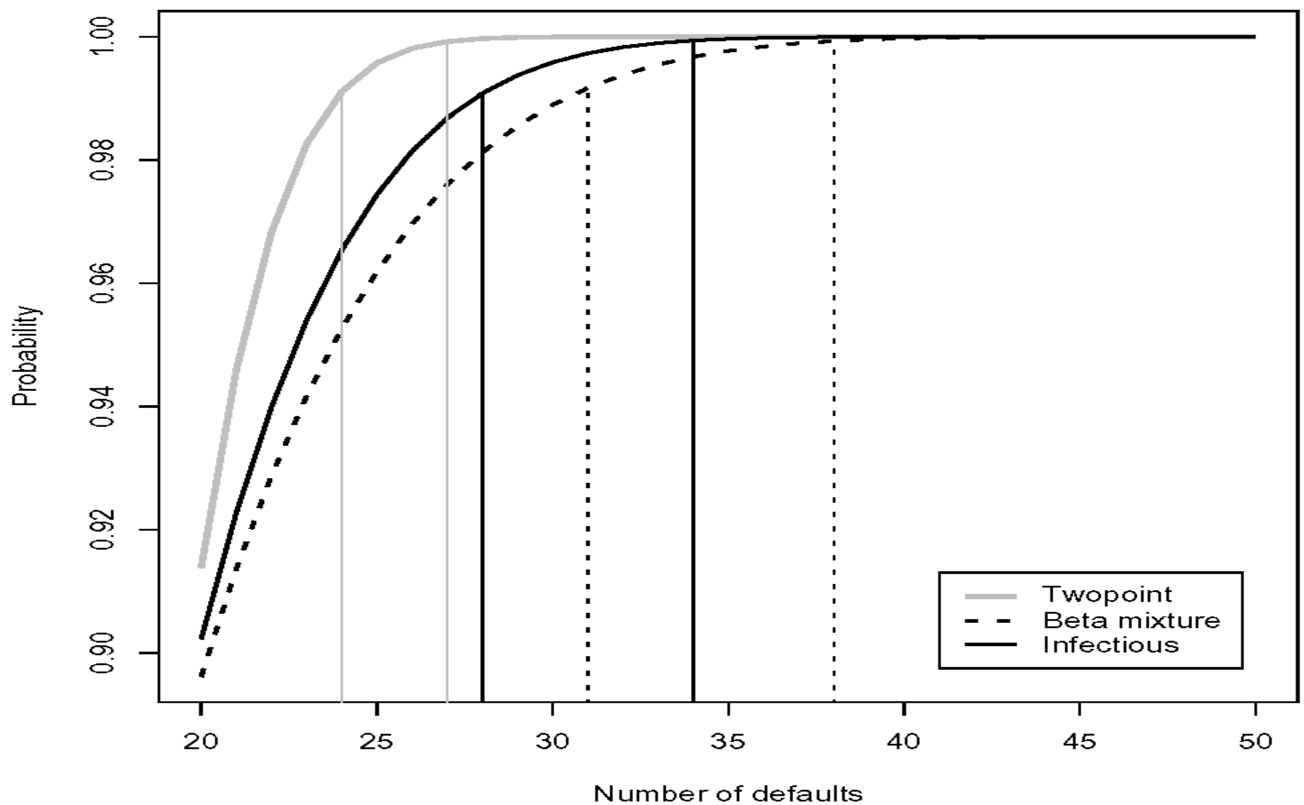
• Buckets and multiple binomial methods
(Dividing issuers into subgroups with different default characteristics)

• Diversity score
(Thinking of the distribution of the number of defaults among 100 (dependent) loans with face value 1 as qpproximated well by (say) D loans with face value 100/D. Match first two moments of loss distribution)

Cifuentes et al. (1998)

Moody's special reports

Moment matching not enough, of course



Dealing with non-homogeneity

Trying to work with inhomogeneity

- Moment generating functions

  (Since the mgf is the product of the individual mgfs, we do not have the problem of treating all constellations of default separately. Problem is accurate inversion, but extends easily to conditional version.)

  Martin, Thompson, Browne (2003)

- Focusing on tail events

  (Capturing the tail behavior of the distribution of defaults)

  Glassermann (2003)

- Simulation

  (Using intensity structure to simplify)

  Duffie and Singleton (2003)

Concluding remarks

- A rich set of models with qualitatively satisfying descriptions
- How strong are dependencies – really?
- Still interest in models with estimable parameters which can capture dependencies
- … and help us understand the structure of risk premia